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TENET: Tail-Event driven NETwork risk

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TENET: Tail-Event driven NETwork risk*

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Abstract

We propose a semiparametric measure to estimate systemic interconnectedness across financial institutions based on tail-driven spill-over effects in a ultra-high dimensional framework. Methodologically, we employ a variable selection technique in a time series setting in the context of a single-index model for a generalized quantile regression framework. We can thus include more financial institutions into the analysis, to measure their interdependencies in tails and, at the same time, to take into account non-linear relationships between them. A empirical application on a set of 200 publicly traded U. S. financial institutions provides useful rankings of systemic exposure and systemic contribution at various stages of financial crisis. Network analysis, its behaviour and dynamics, allows us to characterize a role of each sector in the financial crisis and yields a new perspective of the financial markets at the U. S. financial market 2007 - 2012.

Keywords: Systemic Risk, Systemic Risk Network, Generalized Quantile, Quantile Single-Index Regression, Value at Risk, CoVaR, Lasso

JEL: G01,G18,G32,G38, C21, C51, C63.

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1. Introduction

Systematic risk is often known as risk stemming from the aggregate fluctuations in the economy. The sources of risk are complex, as both exogenous and endogenous factors are involved. This usually calls for a study on a financial network which accounts for interaction between the agents in the financial market. Unlike idiosyncratic risk, the systemic risk is not diversifiable. Although the notion *systemic risk* is not novel in the academic literature (see, e.g, Minsky (1977)), it has been neglected both in the academia and in the financial risk industry until the outbreak of the financial crisis in 2007-2009. The magnitude of repercussions caused by the financial crisis in 2007-2009 and its complexity revealed a significant flaw in financial regulation which has been focused primarily on stability of a single financial institution and triggered several political initiatives across the world such as establishment of Financial Stability Board (FSB) after G-20 London summit in 2009, integration of systemic risk agenda into Basel III in 2010 prior to G-20 meeting in Seoul, enacting the Dodd Frank Wall Street Reform and Consumer Protection Act ('Dodd Frank Act') in U. S. in 2010 which is said to bring the most radical changes into the U. S. financial system since the Great Depression.

These initiatives created several challenges such as identifying *systemically important financial institutions* (SIFIs) whose failure can not only impair the functioning of the financial system but also have adverse effects on the real sector of the economy, studying the propagation mechanism of a shock in a system, or in a network formed by financial institutions, investigating the response of a system to a shock as a whole as well as revealing certain structural patterns in evolution and behavior of a network and establishing a theoretical framework for systemic risk as such.

Although systemic risk is a relatively straightforward concept aimed at measuring risk stemming from interaction between the agents, the variety of risk measures employed at estimating systemic risk and diversity of possible methods to model interaction effects leads to a fact that the literature on this topic is highly heterogenous. The relevant literature in this field can be broadly divided into two groups: economic modelling of systemic risk and financial intermediation including microeconomic (e.g., Beale et al. (2011) and macroeconomic approaches (e.g., Gertler and Kiyotaki (2010) with the emphasis on theoretical, structural framework, and quantitative modelling with the emphasis on empirical analysis. The quantitative literature can be further classified by statistical methodology into quantile regression based modelling such as linear bivariate model by Adrian and Brunnermeier (2011), Acharya et al. (2012), Brownlees and Engle (2012), high-dimensional linear model by Hautsch et al. (2014), partial quantile regression by Giglio et al. (2012) and partial linear model by Chao et al. (2014). Further approaches include principal-component-based analysis, e.g., by Bisias et al. (2012), Rodriguez-Moreno and

Peña (2013) and others; statistical modelling based on default probabilities by Lehar (2005), Huang et al. (2009), and others; graph theory and network topology, e.g., Boss et al. (2006), Chan-Lau et al. (2009).

Our paper belongs the quantitative group of the aforementioned literature, namely, modelling the tail event driven network risk based on quantile regressions augmented with non-linearity and variable selection in ultra-high dimensional time series setting. As a starting point of our research we take co-Value-at-Risk, or CoVaR, model by Adrian and Brunnermeier (2011) (AB), where 'co-' stands for 'conditional', 'contagion', 'co-movement'. To capture the tail interconnectedness between the financial institutions in the system AB evaluate bivariate linear quantile regressions for publicly traded financial companies in the U. S. The CoVaR concept builds upon the concept of VaR with the difference that CoVaR is not simply VaR of an institution itself but is augmented with weighted VaR of another financial institution.

Whereas AB focus on bivariate measurement of tail risk we aim at assessing the tail interconnectedness of a single financial institution with all other financial companies simultaneously. Thus, the primary challenge is selecting the set of relevant risk drivers for each financial institution. Statistically we address this issue by employing the a variable selection method in the context of single-index model for generalized quantile regressions, i.e. for quantiles and expectiles. We further extend it to a time series variable selection context in ultra-high dimensions. The semi-parametric framework due to the single-index model allows us to investigate possible non-linearities in tail interconnectedness. Based on identified relevant risk drivers we construct a financial network based on spill-over effects across financial institutions. Further we propose two indices, namely, systemic contribution index and systemic exposure index where we rank all the companies based on their degree of their contribution (or exposure) to systemic risk.

The assumption of non-linear relationship between returns of financial companies is motivated by previous work by Chao et al. (2014), who find that the dependency between any pair of financial assets is often non-linear, especially in periods of economic downturn. Moreover, non-linearity assumption is more flexible especially in a ultra-high dimensional setting where the system becomes too complex to support the belief of linear relationships.

The model is evaluated based on daily return data on 200 publicly traded U. S. financial institutions from January 1, 2006 till September 1, 2012. The financial institutions are grouped according to their SIC code by industry. The time period from January 1, 2006 to September 1, 2012 covers one recession (2007-2009) and several documented financial crises (2008, 2011). Dividing companies by sectors and including several market perturbations allows not only to select the key players for each time period, but also additionally to highlight the connections between financial industries, which can in turn provide additional information on the nature of market dislocations.

The rest of the paper is organized as follows. In Section 2 our approach to systemic risk modelling is outlined. Section 3 presents the statistical methodology and the related theorems. Section 4 illustrates the empirical application. Section 5 concludes. Appendix A contains proofs and Appendix B contains estimation results.

2. Systemic Risk Modelling

Traditional measures assessing riskiness of a financial institution such as VaR, or expected shortfall (ES) are based either on company characteristics and/or integrate macroprudential variables which account for the general state of the economy. Thus, for example, VaR of a financial institution, a risk measure most commonly used in practice, is defined as

$$\mathrm{P}(-X_{i,t} \geq VaR^{\tau}_{i,t}|D_{i,t}) = \mathrm{P}(X_{i,t} \leq q^{\tau}_{i,t}|D_{i,t}) = \tau, \tag{1}$$

where $X_{i,t}$ is the return of a financial institution, $q^{\tau}_{i,t}$ is the conditional τ-quantile of $X_{i,t}$ at $\tau = (0,1)$ and $D_{i,t}$ denotes the risk drivers relevant for company i, e.g., returns of a financial institution itself, and/or variables reflecting the general state of the economy.

However, recent perturbations at the financial market lead to wide recognition of the fact that in terms of risk what is optimal for a single company is not optimal for the economy as a whole. Therefore, in our model we extend this information set, $D_{i,t}$, and include asset returns of other financial institutions. This allows us to model interaction effects between financial institutions. By extending the information set with asset returns of other financial institutions we assume that the risks are transmitted through returns. This is a simplified assumption, since the risk transmission channels can be defined in a finer way, however, asset returns are known to reflect many aspects of the state of a financial institution simultaneously and, thus, we think it is justified to use them for the analysis of network effects.

The starting point of our research is the AB model proposed. Built upon the concept of VaR they proposed: CoVaR, estimated in two steps. In the first step linear quantile regressions are estimated under standard assumption of $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j|i,t}}(\tau|M_{t-1}, X_{i,t}) = 0$:

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \tag{2}$$

$$X_{j,t} = \alpha_{j|i} + \gamma_{j|i} M_{t-1} + \beta_{j|i} X_{i,t} + \varepsilon_{j|i,t}, \tag{3}$$

where $X_{i,t}$ is the log return of institution i and M_{t-1} are lagged macroprudential variables describing the general state of the economy (See Section 4 for description of macroprudential variables). AB propose to determine VaR of an institution i by regressing log return of company i on macroprudential variables. The obtained $\beta_{j|i}$ in the equation (3) has standard linear regression interpretation, i.e. it determines the sensitivity of log return of an institution j to changes in log return of an institution i. In the second step the CoVaR is calculated by plugging in VaR of company i at level τ into the equation (3):

$$\widehat{\text{VaR}}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, \tag{4}$$

$$\widehat{\text{CoVaR}}^{AB}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\gamma}_{j|i} M_{t-1} + \hat{\beta}_{j|i} \widehat{\text{VaR}}^{\tau}_{i,t}, \tag{5}$$

$$= \widehat{\text{VaR}}^{\tau}_{j|X_i=\widehat{\text{VaR}}^{\tau}_{i,t}} + \hat{\beta}_{j|i} \widehat{\text{VaR}}^{\tau}_{i,t}. \tag{6}$$

Thus, the risk of a financial institution j is estimated as the sum of its own value-at-risk conditional on the fact that the financial institution i is at its VaR level τ and a weighted VaR of an institution i where the magnitude of the weight is determined by the degree of interconnectedness between institutions i and j reflected in $\beta_{j|i}$, or $\beta_{j\leftarrow i}$. By setting j equal to the return on a system, e.g. value-weighted average return on a financial index, and i to the return on a financial company i, we obtain the *contribution* CoVaR which characterizes how a company i influences the rest of the financial system. By doing the reverse, i. e. by setting j equal to a financial institution and i to a financial system, one obtains *exposure* CoVaR, i. e. the extent to which a single institution is exposed to the overall risk of a system.

This approach allows to identify the key elements of systemic risk, namely, network effects, a single institution's contribution to systemic risk and a single institution's exposure to systemic risk, however, it has certain limitations. First of all, it is questionable how to define the return on a system: it has to be proxied by the return on publicly available financial institutions, which, in turn, can be problematic since as AB point out it can create mechanical correlation between a single financial institution and the value-weighted financial index. Although they state that no such correlation is detected, this approach has to be adopted with caution. Secondly, by performing only pairwise quantile regressions one assumes that two companies are interacting in an isolated environment which is not a realistic assumption since all other interaction effects are suppressed.

This motivates us to extend this bivariate model to a (ultra)high dimensional setting by including more variables into the analysis and also allowing for non-linear relationship between the variables. The key element in identifying systemic interconnectedness between the financial institutions lies in precise measurement of the network effects. Based on estimated network effects we evaluate single institution's exposure and single institution's contribution to systemic risk.

To identify the spill-over effects we employ the methodology by Fan et al. (2014) extended to a time series setting. This requires estimation of a single-index model and performing variable selection to identify the key risk drivers for each financial institution simultaneously. More precisely, we estimate:

$$
\begin{aligned}
X_{i,t} &= \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, && (7) \\
X_{i,t} &= g(\beta_{i|-i}^{\top} D_{-i,t}) + \varepsilon_{i,t}, && (8)
\end{aligned}
$$

where $X_{i,t}$ is the log return, $D_{-i,t}$ contains the risk drivers relevant for institution i, $g(\cdot)$ is a link function allowing for the nonlinear relationships. Here $D_{-i,t}$ is equal to macroprudential variables as well as log returns of the financial institutions except for an institution i . We employ the same macroprudential variables as AB, and estimate the VaR by linear quantile regression (7) of log returns of an institution i on macroprudential variables. This is justified by the analysis of Chao et al. (2014), who found no nonlinear effects in regressing $X_{i,t}$ on M_t.

Estimation of equation (8) is performed in two steps: in the first step we perform variable selection to identify relevant risk drivers; in the second step, we estimate link function, $g(\cdot)$, which characterizes relationship between asset returns of an institution i and the rest of the financial system defined in our case as all financial institutions in a sample except for an institution i. Thus, the spill-over effects, for example, from institutions $-i$ to an institution i are determined by coefficients $\beta_{i|-i}$.

We then obtain VaR and CoVaR as follows:

$$
\begin{aligned}
\widehat{\mathrm{VaR}}_{i,t} &= \hat{\alpha}_i + \hat{\gamma}_i M_{t-1}, && (9) \\
\widehat{\mathrm{CoVaR}}_{i|-i,t}^{TENET} &= \widehat{g}(\widehat{\beta}_{i|-i}^{\top} \widehat{D}_{-i,t}^{*}) + \varepsilon_{i,t} && (10)
\end{aligned}
$$

where $\widehat{D}_{-i,t}^{*} = (\mathrm{VaR}_{-i,t}^{*}, M_{t-1})$ and a star denotes that only the VAR of those financial institutions are included which are chosen to be relevant by the variable selection procedure. As one sees in (10) CoVaR comprises not only the influences of financial institutions in a sample except for i, but also incorporates non-linearity reflected in the shape of a link function g. Therefore, we name it $CoVaR^{TENET}$ which stands for Tail-Event-driven NETwork risk.

Non-zero $\widehat{\beta}$ coefficients obtained as a result of estimation of a single-index model allow to measure spill-over effects across the financial institutions and characterize their evolution as a system represented by a *network*. The term *network* refers to a (directed) *graph*, formally written as $G = (V, E)$ where V is a set of vertices and E is a set of links, or edges. To classify financial institutions by their role in the systemic risk context we focus rather on matrix than on network representation of a system and summarize the estimation results in a form of a weighted and unweighted adjacency matrices. A weighted adjacency matrix contains $\widehat{\beta}$ coefficients, see Table 1, whereas an unweighted adjacency matrix is constructed by replacing non-zero $\widehat{\beta}$ coefficients with $d_i = \mathbf{1}\{\widehat{\beta}_i \neq 0\}$.

	$x_{s_{1,1}}$	$\cdots$	$x_{s_{1,n_1}}$	$x_{s_{2,1}}$	$\cdots$	$x_{s_{2,n_2}}$	$\cdots$	$x_{s_{4,1}}$	$\cdots$	$x_{s_{4,n_4}}$	**To**
$x_{s_{1,1}}$	0	$\cdots$	$\widehat{\beta}_{s_{1,n_1}\mid s_{1,1}}$	$\widehat{\beta}_{s_{2,1}\mid s_{1,1}}$	$\cdots$	$\widehat{\beta}_{s_{2,n_2}\mid s_{1,1}}$	$\cdots$	$\widehat{\beta}_{s_{2,1}\mid s_{1,1}}$	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{1,1}}$	$\sum_{j=1}^{p}\widehat{\beta}_{j\mid x_{s_{1,1}}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{s_{1,n_1}}$	$\widehat{\beta}_{s_{1,1}\mid s_{1,n_1}}$	$\cdots$	0	$\widehat{\beta}_{s_{2,1}\mid s_{1,n_1}}$	$\cdots$	$\widehat{\beta}_{s_{2,n_2}\mid s_{1,n_1}}$	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{1,n_1}}$	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{1,n_1}}$	
$x_{s_{2,1}}$	$\widehat{\beta}_{s_{1,1}\mid s_{2,1}}$	$\cdots$	$\widehat{\beta}_{s_{1,1}\mid s_{2,1}}$	0	$\cdots$	$\widehat{\beta}_{s_{2,n_2}\mid s_{2,1}}$	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{2,1}}$	$\cdots$	$\widehat{\beta}_{s_{4,n_4}\mid s_{2,1}}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{s_{2,n_2}}$	$\widehat{\beta}_{s_{1,1}\mid s_{2,n_2}}$	$\cdots$	$\widehat{\beta}_{s_{1,n_1}\mid s_{2,n_2}}$	$\widehat{\beta}_{s_{2,1}\mid s_{2,n_2}}$	$\cdots$	0	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{2,n_2}}$	$\cdots$	$\widehat{\beta}_{s_{4,n_4}\mid s_{2,n_2}}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{s_{4,1}}$	$\widehat{\beta}_{s_{1,1}\mid s_{4,1}}$	$\cdots$	$\widehat{\beta}_{s_{1,n_1}\mid s_{4,1}}$	$\beta_{s_{1,n_1}\mid s_{4,1}}$	$\cdots$	$\widehat{\beta}_{s_{2,n_2}\mid s_{4,1}}$	$\cdots$	0	$\cdots$	$\widehat{\beta}_{s_{4,n_4}\mid s_{4,1}}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$x_{s_{4,n_4}}$	$\widehat{\beta}_{s_{1,1}\mid s_{4,n_4}}$	$\cdots$	$\widehat{\beta}_{s_{1,n_1}\mid s_{4,n_4}}$	$\beta_{s_{2,1}\mid s_{4,n_4}}$	$\cdots$	$\widehat{\beta}_{s_{2,n_2}\mid s_{4,n_4}}$	$\cdots$	$\widehat{\beta}_{s_{4,1}\mid s_{4,n_4}}$	$\cdots$	0	
From	$\sum_{i=1}^{p}\widehat{\beta}_{x_{s_{1,1}}\mid i}$	$\cdots$			$\cdots$		$\cdots$		$\cdots$		$\sum_{i=1}^{p}\sum_{j=1}^{p}\widehat{\beta}_{i\mid j}$

Table 1: An adjacency matrix for financial institutions classified according sectors denoted as $s_{m,k}$ where $m = 1, \cdots, 4$ are sectors and $k = 1, \cdots, n_m$ are the corresponding samples in a sector.

The above $p \times p$ matrix in Table 1 where p is equal to the number of financial institutions represents total connectedness across variables at each time point $t = 1, \cdots, n$. The adjacency matrix, or a total connectedness matrix, is sparse and off-diagonal since our model by construction does not allow for self-loop effects (namely one variable cannot be regressed on itself). The rows of this matrix correspond to outgoing edges for a variable in a respective row and the columns correspond to incoming edges for a variable in a respective column. For example, row 1 contains $\widehat{\beta}$ coefficients of all the variables that are influenced by company 1 from sector 1, $x_{s_{1,1}}$. Thus, a cell $(x_{s_{1,1}}, x_{s_{1,n_1}})$ contains $\widehat{\beta}_{s_{1,n_1}|s_{1,1}}$ which is the magnitude of the influence of company 1 from sector 1, $x_{s_{1,1}}$, on company n from sector 1, $x_{s_{1,n_1}}$.

To classify institutions according to their systemic importance we construct two indices based on the weighted adjacency matrix: index of systemic contribution, $C_{i,t}$, and index of systemic exposure, $E_{i,t}$. This allows us to identify risk emitters and risk recipients. We define the index of systemic contribution, $C_{i,t}^{m,k}$, as a proportion of the sum of $\widehat{\beta}$ coefficients corresponding to outgoing links, or a sum of each row of a weighted adjacency matrix, to the total sum of $\widehat{\beta}$ coefficients at the particular $t = 1, \cdots, n$:

$$C_{i,t}^{m,k} \stackrel{\text{def}}{=} \left(1 + \frac{\sum_{j \neq s_{m,k}} \widehat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^{p} \sum_{j=1}^{p} \widehat{\beta}_{i|j}}\right) \widehat{\text{VaR}}_{i,t}^{m,k}, \tag{11}$$

where $m = 1, \cdots, 4$ are sectors and $k = 1, \cdots, n_m$ are the sample sizes of corresponding sectors. Index of systemic contribution weights VaR of a corresponding financial institution with proportion of its influence on the rest of the financial companies. In other words, VaR of a financial institution is proportional to its monopoly power at the market defined through its systemic connectedness.

Correspondingly, we define the index of systemic exposure, $E_{i,t}^{m,k}$, as a proportion of the sum of $\widehat{\beta}$ coefficients corresponding to incoming links, or sum of each column of a weighted adjacency matrix, to the total sum of $\widehat{\beta}$ coefficients at the particular $t = 1, \cdots, T$:

$$E_{i,t}^{m,k} \stackrel{def}{=} \left(1 + \frac{\sum_{i \neq s_{m,k}} \widehat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^{p} \sum_{j=1}^{p} \widehat{\beta}_{i|j}}\right) \widehat{\text{VaR}}_{i,t}^{m,k}, \tag{12}$$

The conceptual difference to AB is that whereas they assume that CoVaR of a financial institution increases if it is influenced by another institution, we assume that CoVaR increases if a financial institution is not only being influenced but also exerts significant influence on other financial institutions itself. This is motivated by the fact that not only heavily influenced institutions become riskier in the period of crisis, but also those with high degree of monopoly power in terms of systemic interconnectedness are potentially the first ones to fail. After constructing these indices we classify a financial institution either as a risk-emitter, or as a risk-recipient by computing $\max\{C_{i,t}^{m,k}, E_{i,t}^{m,k}\}$. As a example, the value-at-risk based on exposure and contribution indices for selected companies, Fairfield Greenwich Group (FGG, Broker-Dealers), FBL Financial Group (FFG, Insurance), which are the largest risk-contributors and risk-receivers are presented in Figures 11 and 12. A more comprehensive empirical analysis can be found in Section 4.

3. Statistical Methodology

Let us denote $X_t \in \mathbf{R}^p$ as p as the variables $D_{-i,t}$ before, p can be very large, namely of exponential rate. We also suppress the subscripts of the coefficients $\beta_{i|j}$s, as we focus on one regression. The SIM of (8) is defined to be:

$$Y_t = g(X_t^\top \beta^*) + \varepsilon_t, \tag{13}$$

where $\{X_t, \varepsilon_t\}$ are strong mixing processes. Regressors X_ts can be the lagged variables of Y_t.

Note that (13) can be formulated in a location model and identified in a quasi maximum likelihood framework: the direction β (for known $g(\cdot)$) is the solution of

$$\min_\beta \mathsf{E}\, \rho_\tau\{Y_t - g(X_t^\top \beta)\}, \tag{14}$$

with loss function

$$\rho_\tau(u) = \rho(u) = \tau u \mathbf{1}(u > 0) + (1-\tau) u \mathbf{1}(u < 0), \tag{15}$$

$$\mathsf{E}(\psi_\tau\{Y_t - g(X_t^\top \beta)\}|X_t) = 0 \quad a.s.$$

(where $\psi_\tau(\cdot)$ is the derivative (a subgradient) of $\rho_\tau(\cdot)$). It can be reformulated as $F^{-1}_{\varepsilon|X_t}(\tau) = 0$.

The model is similar to the location scale model considered in Franke et al. (2014). Note that it is not hard to extend it to a quantile AR-ARCH type of single index model,

$$Y = g(X_t^\top \beta) + \sigma(X_t^\top \gamma)\varepsilon_t \tag{16}$$

To estimate the shape of a link function $g(\cdot)$ and selected β coefficients we adopt minimum average contrast estimation approach (MACE) outlined in Fan et al. (2014). The estimation of β and $g(\cdot)$ is as following:

$$
\begin{aligned}
\hat{\beta}, \hat{g}(\cdot) &\stackrel{\mathrm{def}}{=} \arg\min -L_n(\beta) \\
&= \arg\min n^{-1} \sum_{j=1}^{m} \sum_{t=1}^{n} \rho_\tau \left\{ Y_t - g(\beta^\top X_t) - g'(\beta^\top X_t)\beta^\top (X_t - X_j) \right\} \\
&\quad K_h\{\beta^\top (X_t - X_j)\} / \sum_{t=1}^{n} K_h(X_{tj}^\top \beta), \qquad (17)
\end{aligned}
$$

where $K_h(\cdot) = h^{-1}K(\cdot/h)$, $K(\cdot)$ is a kernel e.g. Gaussian kernel and h is a bandwidth. Since the data is not equally spaced we choose a bandwidth h based on k-nearest neighbor procedure (See Härdle et al. (2004)). The optimal k, number of neighbors, are selected based on a cross-validation criterion. The implementation involves an iteration between estimating β and $g(\cdot)$, with a consistent initial estimate for β, see for example Wu et al. (2010). Further in our procedure, we argument the estimation problem in equation (17) with variable selection to estimate β:

$$
\hat{\beta}_\tau, \hat{g}(\cdot) = \arg\min_{\beta, g(\cdot)} n^{-1} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho_\tau \{ X_t - g(\beta^\top X_j) - g'(\beta^\top X_j) X_{tj}^\top \beta \} \omega_{tj}(\beta) + \sum_{l=1}^{p} \gamma_\lambda(|\beta_l|^\theta), \qquad (18)
$$

where $X_{tj} = X_t - X_j$, $\omega_{tj}(\beta) \stackrel{\mathrm{def}}{=} \dfrac{K_h(X_{tj}^\top \beta)}{\sum_{t=1}^{n} K_h(X_{tj}^\top \beta)}$, $\theta \geq 0$, and $\gamma_\lambda(t)$ is some non-decreasing function concave for $t \in [0, +\infty)$ with a continuous derivative on $(0, +\infty)$. We now discuss the choices of an optimal penalty function and an optimal penalization parameter. There are several approaches in the literature concerning choosing penalty function. These approaches can be classified based on the properties desirable for an optimal penalty function, namely, unbiasedness, sparsity and continuity (Fan and Li (2001)). The classical approach known as least absolute shrinkage and selection operator (LASSO) is proposed for mean regression by Tibshirani (1996), which is based on L_1 penalty for the coefficients. Numerous studies further adapt LASSO variable selection procedure to a quantile regression framework such as Yu et al. (2003), Li and Zhu (2008), Belloni and Chernozhukov (2011), etc. While achieving sparsity, L_1-norm penalty tends to over-penalize the large coefficients as the LASSO penalty increases linearly in the magnitude of its argument, and, thus, may introduce large bias to estimation. As a remedy to this problem adaptive LASSO estimation procedure has been proposed (Zou (2006); Zheng et al. (2013)). Another approach to alleviate bias stemming from LASSO procedure is proposed by Fan and Li (2001) known as Smoothly Clipped Absolute Deviation (SCAD):

$$\gamma_\lambda(t) = \begin{cases} \lambda|t| & \text{for } |t| \le \lambda, \\ -(t^2 - 2a\lambda|t| + \lambda^2)/2(a-1) & \text{for } \lambda < |t| \le a\lambda, \\ (a+1)\lambda^2/2 & \text{for } |t| > a\lambda, \end{cases}$$

where $\lambda > 0$ and $a > 2$. Fan and Li (2001) recommend to use $a = 3.7$.

As for selecting λ, there are two common ways: data-driven generalized cross-validation criterion (GCV) (Fan and Li (2001)) and likelihood-based Schwartz, or Bayesian information criterion-type criteria (SIC, or BIC) (Schwarz (1978); Koenker et al. (1994)), and their further modifications. The most commonly used criterion is GCV (Fan and Li (2001), Tibshirani (1996)), however, it has been shown that it leads to an overfitted model (Wang et al. (2007)). Therefore, we primarily employ a modified BIC-type model selection criteria proposed by Wang et al. (2007) and use GCV criterion only to verify whether GCV and BIC diverge significantly.

Define $\hat{\beta}_\tau \stackrel{\text{def}}{=} (\hat{\beta}_{\tau(1)}^\top, \hat{\beta}_{\tau(2)}^\top)^\top$ as the estimator for $\beta^* \stackrel{\text{def}}{=} (\beta_{(1)}^{*\top}, \beta_{(2)}^{*\top})^\top$ attained by the loss in (18). Let $\hat{\beta}_{\tau(1)}$ and $\hat{\beta}_{\tau(2)}$ be the first q components and the remaining $p-q$ components of $\hat{\beta}_\tau$ respectively. If in the iterations, we have the initial estimator $\hat{\beta}_{(1)}^{(0)}$ as a $\sqrt{n/q}$ consistency one for $\beta_{(1)}^*$, we will obtain with a very high probability, an oracle estimator of the following type, say $\tilde{\beta}_\tau = (\tilde{\beta}_{\tau(1)}^\top, \mathbf{0}^\top)^\top$, since the oracle knows the true model $\mathcal{M}_* \stackrel{\text{def}}{=} \{l : \beta_l^* \neq 0\}$. The following theorem shows that the penalized estimator enjoys the oracle property. Define $\hat{\beta}^0$ as the minimizer with the same loss in (18) but within subspace $\{\beta \in \mathbb{R}^p : \beta_{\mathcal{M}_*^c} = \mathbf{0}\}$. We have the following conditions needed for theorems.

Condition 1. The kernel $K(\cdot)$ is a continuous symmetric function. The link function $g(\cdot) \in C^2$.

Condition 2. The loss function $\rho_\tau(x)$ is convex and $\psi_\tau(x)$, the derivative (or a subgradient) of $\rho_\tau(x)$, satisfies $\mathsf{E}\,\psi_\tau(\varepsilon_t) = 0$ and $\inf_{|v| \le c} \partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t - v) = C_1$ where $\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t - v)$ is the partial derivative with respect to v, and C_1 is a constant.

Condition 3. The density of $\beta^{*\top} X$ is bounded with bounded absolute continuous first-order derivatives on its support. Assume $\mathsf{E}\{\psi_\tau(\varepsilon|X)\} = 0$ a.s., which means for a quantile loss we have $F_{\varepsilon|X}^{-1}(\tau) = 0$. Let $X_{t(1)}$ denote the sub-vector of X_t consisting of its first q elements. Let $Z_t \stackrel{\text{def}}{=} X_t^\top \beta^*$ and $Z_{tj} \stackrel{\text{def}}{=} Z_t - Z_j$. Define $C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\,\mathsf{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$, and
$C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$ and the matrix $C_{1(1)}$ satisfies $0 < L_1 \le \lambda_{\min}(C_{0(1)}) \le \lambda_{\max}(C_{0(1)}) \le L_2 < \infty$ for positive constants L_1 and L_2. There exists a constant $c_0 > 0$ such that $\sum_{t=1}^n \{\|X_{t(1)}\|/\sqrt{n}\}^{2+c_0} \to 0$, with $0 < c_0 < 1$. $v_{tj} \stackrel{\text{def}}{=} Y_t - a_j - b_j X_{tj}^\top \beta$. Also, exists a constant C_3 such that for all β close

to β^* ($\|\beta - \beta^*\| \leq C_3$)

$$\| \sum_t \sum_j X_{(0)tj} \omega_{tj} X_{(1)tj}^\top \partial \, \mathsf{E} \, \psi_\tau(v_{tj}) \|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1}).$$

Condition 4. The penalty parameter λ is chosen such that $\lambda = \mathcal{O}(n^{-1/2})$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \scriptstyle{\mathcal{O}}(n^{\alpha_1 - \alpha_2/2}\lambda) = \scriptstyle{\mathcal{O}}(n^{-1/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \to 0$ and $h^{-1}\sqrt{q/n} = \scriptstyle{\mathcal{O}}(1)$ as n goes to infinity, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}\{\exp(n^\delta)\}$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$.

Condition 5. The error term ε_t satisfies $\mathsf{Var}(\varepsilon_t) < \infty$. Assume that

$$\sup_t \mathsf{E}\big|\psi_\tau^m(\varepsilon_t)/m!\big| \leq s_0 M^m$$
$$\sup_t \mathsf{E}\big|\psi_\tau^m(x_{tj})/m!\big| \leq s_0 M^m$$

where s_0 and M are constants, and $\psi_\tau(\cdot)$ is the derivative (a subgradient) of $\rho_\tau(\cdot)$.

Condition 6. The conditional density function $f(\varepsilon|Z_t = u)$ is bounded and absolutely continuous differentiable.

Condition 7. The link function $g(\cdot)$ satisfies a Lipschitz condition in the support of $\beta^\top X$:

$$|g(z) - g(\tilde{z})| \leq C|z - \tilde{z}| \tag{19}$$

Conditions 8. $\{X_{tj}, \varepsilon_t\}_{t=-\infty}^{t=\infty}$ are strong mixing process for any j. Moreover, there exists positive constants c_{m1} and c_{m2} such that the $\alpha-$ mixing coefficient for every $j \in \{1, \cdots, p\}$,

$$\alpha(l) \leq \exp(-c_{m1} l^{c_{m2}}), \tag{20}$$

where $c_{m2} > 2\alpha$.

With all the above definitions and conditions, we can derive the following theorems.

THEOREM 3.1. *Under Conditions 1-8, the estimators $\hat{\beta}^0$ and $\hat{\beta}_\tau$ exist and coincide on a set with probability tending to 1. Moreover,*

$$\mathrm{P}(\hat{\beta}^0 = \hat{\beta}_\tau) \geq 1 - (p - q)\exp(-C' n^\alpha) \tag{21}$$

for a positive constant C'.

THEOREM 3.2. *Under Conditions 1-8, we have*

$$\|\hat{\beta}_{\tau(1)} - \beta^*_{(1)}\| = \mathcal{O}_p\{(D_n + n^{-1/2})\sqrt{q}\} \tag{22}$$

For any unit vector $\boldsymbol{b}$ *in* $\mathbb{R}^q$, *we have*

$$\boldsymbol{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} \sqrt{n}(\hat{\beta}_{\tau(1)} - \beta^*_{(1)}) \xrightarrow{\mathcal{L}} \mathbb{N}(0,1) \tag{23}$$

where recall that $C_{1(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\mathsf{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\}[g'(Z_t)]^2[\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}][\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}]^\top\}$, *and* $C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$. *Note that* $\mathsf{E}(X_{(1)}|Z_t)$ *denotes a* $p \times 1$ *dimension vector with* j*th element* $\mathsf{E}(X_{j(1)}|Z_t)$, $j = 1, \cdots, q$, *and* $Z_t \stackrel{\text{def}}{=} X_t^\top \beta^*$, $\psi_\tau(\varepsilon)$ *is a choice of the subgradient of* $\rho_\tau(\varepsilon)$ *and* $\sigma_\tau^2 \stackrel{\text{def}}{=} \mathsf{E}[\psi_\tau(\varepsilon_t)]^2/[\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t)]^2$, *where*

$$\partial\,\mathsf{E}\,\psi_\tau(\cdot)|Z_t = \frac{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t - v)^2|Z_t}{\partial v^2}\Big|_{v=0}. \tag{24}$$

Let us now look at the distribution of $\hat{g}(\cdot)$ and $\hat{g}'(\cdot)$, the estimator of $g(\cdot)$, $g'(\cdot)$.

THEOREM 3.3. *Under Conditions 1-8, let* $\mu_j \stackrel{\text{def}}{=} \int u^j K(u)du$ *and* $\nu_j \stackrel{\text{def}}{=} \int u^j K^2(u)du$, $j = 0, 1, 2$. *For any interior point* $z = x^\top \beta^*$, $f_Z(z)$ *is the density of* Z_t, $t = 1, \ldots, n$, *if* $nh^3 \to \infty$ *and* $h \to 0$, *we have*

$$\sqrt{nh}\sqrt{f_Z(z)/(\nu_0\sigma_\tau^2)}\left\{\widehat{g}(x^\top\widehat{\beta}) - g(x^\top\beta^*) - \frac{1}{2}h^2 g''(x^\top\beta^*)\mu_2\partial\,\mathsf{E}\,\psi_\tau(\varepsilon)\right\} \xrightarrow{\mathcal{L}} \mathbb{N}(0,\ 1),$$

Also, we have

$$\sqrt{nh^3}\sqrt{\{f_Z(z)\mu_2^2\}/(\nu_2\sigma_\tau^2)}\left\{\widehat{g'}(x^\top\widehat{\beta}) - g'(x^\top\beta^*)\right\} \xrightarrow{\mathcal{L}} \mathbb{N}(0,\ 1),$$

The dependence doesn't have any impact on the rate of the convergence of our nonparametric link function. As the degree of the dependence is measured by the mixing coefficient α, is weak enough such that Condition 8 is satisfied. This is also in line with the results in Kong et al. (2010). In fact we assume exponential decaying rate here, which implies the (A.4) in Kong et al. (2010).

4. Empirical Analysis

4.1. Data

Our analysis focuses on the panel of 200 publicly traded U. S. financial institutions between January 1, 2006 and September 1, 2012 corresponding to SIC code from 6000 to 6799. SIC codes are used to divide companies into the following sectors: (1) depositories, (2) insurance companies, (3) broker-dealers, (4) others, see Table 2 in Appendix B for a complete list.

Due to its high-dimensionality this dataset approximates the aggregate fluctuations in the system fairly well. Thus we do not include companies operating in the real sector and do not investigate linkages between financial and real sectors of the economy. We also do not include any proxy for the shadow banking sector which may play a role in the 2007-2009 crisis. The time period from January 1, 2006 till September 1, 2012 covers one recession (2007-2009) and several financial crises (2008, 2011).

Our analysis is based on the daily returns of the above mentioned financial institutions (Table 2). Market returns are a rich and robust source of information reflecting the overall state of the company. Apart from the data on the financial companies we use daily observations of macroprudential variables which characterize the general state of the economy. These variables are defined as follows: (i) the implied volatility index, VIX, reported by the Chicago Board Options Exchange; (ii) short term liquidity spread denoted as the difference between the three-month repo rate and the three-month bill rate to measure short-term liquidity risk; (iii) the changes in the three-month Treasury bill rate from the Federal Reserve Board; (iv) the changes in the slope of the yield curve corresponding to the yield spread between the ten-year Treasury rate and the three-month bill rate form the Federal Reserve Board; (v) the changes in the credit spread between BAA-rated bonds and the Treasury rate; (vi) the weekly equity market returns from CRSP, and (vii) the returns on a real estate sector in excess of the market returns.

4.2. Estimation Results

Empirical analysis is performed at three levels: first of all, we characterize the behavior of a system as a whole, in the second step, we investigate fluctuations across the financial sectors, and at last, we analyze the systemic importance of particular companies. Figure 1 presents the risk network estimated for the window starting at 20070111, and Figure 2 considers the sparsified network with further thresholding.

To describe aggregate fluctuations in a system represented by financial institutions in a sample we define total connectedness in terms of $\widehat{\beta}$ coefficients, $TC^s \stackrel{\text{def}}{=} \sum_{i=1}^{p} \sum_{j=1}^{p} |\widehat{\beta}_{i|j}|$, and number of links, $TC^n \stackrel{def}{=} \sum_{i=1}^{p} \sum_{j=1}^{p} d_{i|j} \stackrel{\text{def}}{=} \sum_{i=1}^{p} \sum_{j=1}^{p} \mathbf{1}\{\widehat{\beta}_{i|j} \neq 0\}$. We distinguish between the aggregate magnitude of $\widehat{\beta}_{i|j}$ coefficients and total number of links at each time point to account for the periods when there are large number of small links, and/or few significant links. Figure 3 shows that the beginning of 2007-2009 financial crisis is characterized by smaller magnitude of $\widehat{\beta}_{i|j}$ coefficients and fewer links. As the crisis was unfolding, the system became more heavily interconnected and reached its peak in the second quarter of 2008 and lasted until beginning of 2009. Total connectedness across financial institutions started to decrease in the second quarter of 2009 and reached it minimum by the first half of 2010. Several negative shocks at the U. S. financial market contributed to a slow increase both in TC^s and TC^n from the second quarter of 2010 to through 2011. Examples are Flash Crash in May 2010 attributed to the U. S. reaction to the debt crisis in Greece and US debt-ceiling crisis in July 2011. The fluctuations in linkages in 2010-2012 were much smaller than during the financial crisis due to overall positive performance of the markets.

The average values for a penalization parameter $\widehat{\lambda}$ for 200 financial companies are presented in Figure 3 by a dotted line. One sees that the estimation results of total connectivity measures, TC^s and TC^n have the same time trend as $\widehat{\lambda}$. The penalization parameter $\widehat{\lambda}$ determines the number of selected risk drivers in the whole procedure, and the fluctuations in $\widehat{\lambda}$ can also document the noise level and interconnections of the whole system.

Figure 3 plots also $\widehat{\lambda}$ empirically. While staying at a low level in the beginning of 2007, $\widehat{\lambda}$ increases together with TC^s and TC^n. Both TC^s and average $\widehat{\lambda}$ values reach its maximum approximately in the middle of 2009, which is during the period of subprime crisis. As the economy recovers the average $\widehat{\lambda}$, TC^s and TC^n values all start to fall, which suggest a lessened comovement across the financial institutions. Market disturbances in 2011 corresponding to European debt crisis are reflected by another smaller bump TC^s and TC^n.

We now turn to analyzing the relative contribution and exposure of each sector as well as interconnectedness across sectors. To obtain a measure of linkages per company in each sector, we compute the following density measures for each sector in terms of exposure and contribution: $D_{exp}^m \stackrel{\text{def}}{=} \frac{1}{n_m} \sum_{i=1}^{p} \sum_{j=1}^{n_m} |\widehat{\beta}_{s_m,j|-i}|$ and $D_{contr}^m \stackrel{\text{def}}{=} \frac{1}{n_m} \sum_{j=1}^{p} \sum_{i=1}^{n_m} |\widehat{\beta}_{-j|s_m,i}|$ where m is fixed and n_m is the sample size of a corresponding sector. By dividing n_m we summarize variations of the relative intensity per link within each sector. The relative exposure D_{exp}^m does not exhibit great fluctuations across time as shown in Figure 4, except that one observes a slightly decreasing intensity for the depositories and a large increase in exposure intensity of the sector "Others" from 2010 to 2011. This significant increase is possibly due to the heterogeneity in sector "Others" which contains companies such as, American Express, (a credit card company), Equifax, (a credit-rating agency), Sterling Financial Corporation, (a consumer and commercial bank acquired by PNC Financial Services Co. in 2007). In contrast to D^m_{exp}, the contribution density D^m_{contr} (displayed in Figure 5) is more volatile, see Figure 5. The contribution densities of depositories and insurance companies are relatively stable with the former dominating the latter in the given time period 2007-2012. However, one can observe a interesting interplay in contribution densities between depositories and insurance companies in 2007-2008 when insurance companies overtook leadership over the depositories for a short time during the crisis. Contribution density of broker-dealers exhibits seasonal-type fluctuations. Broker-dealers played a significant role in the financial crisis as suggested by their relatively higher contribution density. This is justifiable as they may be more susceptible to runs and bankruptcy due to their tendency to use short term credit markets to finance their operations. This was also the reason why during the crisis this particular sector needed extensive support from the government in order to prevent complete market break-down.

We further describe connections between different financial sectors by defining interconnectedness measures as follows: $IC_{exp} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n_{m1}} \sum_{j=1}^{n_{m2}} \widehat{\beta}_{i|j}}{\sum_{i=1}^{p} \sum_{j=1}^{p} \widehat{\beta}_{i|j}}$ and $IC_{contr} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n_{m1}} \sum_{j=1}^{n_{m2}} \widehat{\beta}_{j|i}}{\sum_{i=1}^{p} \sum_{j=1}^{p} \widehat{\beta}_{i|j}}$. In other words, to compute how depositories were influenced by, e.g., insurance sector, we sum up $\widehat{\beta}_{i|j}$ in rows of the adjacency matrix corresponding to insurance sector and columns corresponding to depositories. The results for this network statistics are presented on Figure 6. One can see on Figure 6, rows 2, 3 and 4, that depositories dominate in the interconnectedness between the institutions for all sectors. Especially significant linkages both in terms of exposure and contribution exist between depositories and insurance sector. This pattern changes only around 2011 when broker-dealers influenced the banking sector more heavily than other sectors. While being the most exposed and most "contributing" sector among others broker-dealers unlike insurance sector has more balanced structure of interconnectedness with remaining three sectors.

Based on the above analysis we have the following conclusions: (1) the connections between institutions tend to increase both in terms of magnitude of coefficients and the number of links before the financial crisis, (2) the network is characterized by numerous heavy links at the peak of a crisis, (3) the connections between institutions reflected by magnitude of coefficients and the number of links get weaker as the financial system stabilized, (4) the in-going links, or exposure, is far less volatile than the out-going links, or contribution. Whereas banks and insurance exhibit relatively stable contribution pattern, the "Others" and, especially, broker-dealer sectors are highly volatile. While 2008 was dominated by the banks and/insurance, the second half and the aftermath of a financial crisis are characterized by increased contribution of broker-dealers. A significant increase in the magnitude of coefficients due to broker-dealer sector is observed also around 2011.

After having summarized network measures at the aggregate and sector level we turn to analysis of a network at a single companies level. We define in- and out- degrees of one variable as follows: $c_{d^-}^{m,k} \stackrel{\text{def}}{=} \sum_{i=1}^{p} |\widehat{\beta}_{s_{m,k}|-i}|$ and $c_{d^+}^{m,k} \stackrel{\text{def}}{=} \sum_{j=1}^{p} |\widehat{\beta}_{-j|s_{m,k}}|$. We further aggregate the two measure over all windows for each company. As for total connectedness we calculate both measures based on adjacency matrices.

The estimation results for in- and outdegree are presented in Figures 7 and 8. The most significant risk-recipients and risk-emitters are summarized in Tables 3 to 8. The distribution of incoming and outgoing links differ significantly: while judged by the number of incoming links almost all financial institutions are equally influenced by others except for Wisdom Tree Investments (WETF) having the most in-gong links (5584), the structure of out-going is more heterogenous with multiple companies dominating over the others. Based on the ratio of the total sum of β coefficients to the total number of links for each company, it is possible to identify the most influential financial institutions. In case for IN links these institutions are Fairfield Greenwich Group (FGG, Broker-Dealers), FBL Financial Group (FFG, Insurance), Radian Group (RDN, Insurance), Morgan Stanley (MS, Broker-Dealer), Pinnacle Financial Partners (PNFP, Depositories), and Hartford Financial Services Group (HIG, Insurance). Ratio ranking results suggests the general importance of broker-dealer industry in the whole financial system especially during financial crisis.

Whereas distribution of IN links allows to identify the key players obviously, it is not easy to identify dominating companies with OUT links. Based on the ranking of sum of $c_{d^+}^{m,k}$ the five most influential companies are FBL Financial Group (FFG, Insurance), Janus Capital (JNS, Others), Lincoln National Corporation (LNC, Insurance), Morgan Stanley (MS, Broker-Dealer), and Fairfield Greenwich Group (FGG, Broker-Dealers). However, Janus Capital's links are so dispersed (10351) that it has very low ratio of sum to number of links. The same holds true for Lincoln National Corporation and Morgan Stanley. Thus, companies with the highest loads per OUT link are Fairfield Greenwich Group (FGG, Broker-Dealers) and FBL Financial Group (FFG, Insurance), Loews Corporation (L, Insurance) comes as the third one, followed by Arthur J Callagher & Co (AJG, Insurance) and BlackRock Inc. (BLK, Others). Unlike with IN links where both Fairfield Greenwich Group (FGG, Broker-Dealers) and FBL Financial Group (FFG, Insurance) share the leadership, for OUT links Fairfield Greenwich Group (FGG, Broker-Dealers) is clearly the only leader with 0.99 ratio.

Thus, in terms of systemic contribution the IN links are primarily dominated by broker-dealers and insurance sectors, whereas OUT links are primarily due to either broker-dealers or banks sectors. Other sectors have approximately equal share in terms of both

IN and OUT links. This is reasonable as it is likely that insurance companies were connected with the most of the companies to hedge against possible risk. However, insurance companies themselves were not the active players at the market, and were dominated by broker-dealers and/or banks in terms of OUT links.

Further, we examine the shape of the link function in the crisis period as well as in the period of relative financial stability. As one can observe in Figure 9 the link function tends to be almost linear in a financial crisis period. It exhibits non-linearity in a stable period as displayed in Figure 10.

Finally, we construct the indices of systemic exposure and systemic contribution, the results for the whole sample are presented in Table 9. (Cumulative Contribution is defined as summing up $\frac{\sum_{j\neq s_{m,k}} \widehat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^{p}\sum_{j=1}^{p}\widehat{\beta}_{i|j}}$ over all windows and Exposure Indices is defined as summing up $\frac{\sum_{i\neq s_{m,k}} \widehat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^{p}\sum_{j=1}^{p}\widehat{\beta}_{i|j}}$ over all windows.)

5. Conclusion

In this paper we propose a semiparametric framework to assess systemic importance of financial institutions based on their interconnectedness in tails. We use a semiparametric model to allow for more flexible modeling of relationship between the variables. This is especially justified in a ultra-high-dimensional setting when the assumption of linearity is not likely to hold. In order to face these challenges statistically we estimate a single-index model in a generalized quantile regression framework while simultaneously performing variable selection. Ultra-high dimensional setting allows us to include more variables into the analysis.

Our empirical results show that there is growing interconnectedness during the period of a financial crisis, and network-based measure reflecting the connectivity between companies can be used to forecast the market disturbances. Moreover, by including more variables into the analysis we can investigate the overall performance of different financial sectors, depositories, insurance, broker-dealers, and others. We base our analysis on the network measures. Estimations results show relatively high importance of broker-dealers industry in the financial crisis. We also observe strong non-linear relationship between the variables, especially, in the period of relative financial stability.

6. Appendix A: Proof

Condition 1. The kernel $K(\cdot)$ is a continuous symmetric function. The link function $g(\cdot) \in C^2$.

Condition 2. The loss function $\rho_\tau(x)$ is convex and $\psi_\tau(x)$, the derivative (or a subgradient) of $\rho_\tau(x)$, satisfies $\mathsf{E}\,\psi_\tau(\varepsilon_t) = 0$ and $\inf_{|v|\le c} \partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t - v) = C_1$ where $\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t - v)$ is the partial derivative with respect to v, and C_1 is a constant.

Condition 3. The density of $\beta^{*\top}X$ is bounded with bounded absolute continuous first-order derivatives on its support. Assume $\mathsf{E}\{\psi_\tau(\varepsilon|X)\} = 0$ a.s., which means for a quantile loss we have $F^{-1}_{\varepsilon|X}(\tau) = 0$. Let $X_{t(1)}$ denote the sub-vector of X_t consisting of its first q elements. Let $Z_t \stackrel{\text{def}}{=} X_t^\top\beta^*$ and $Z_{tj} \stackrel{\text{def}}{=} Z_t - Z_j$. Define $C_{0(1)} \stackrel{\text{def}}{=}$ $\mathsf{E}\,\mathsf{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$, and
$C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$ and the matrix $C_{1(1)}$ satisfies $0 < L_1 \le \lambda_{\min}(C_{0(1)}) \le \lambda_{\max}(C_{0(1)}) \le L_2 < \infty$ for positive constants L_1 and L_2. There exists a constant $c_0 > 0$ such that $\sum_{t=1}^n \{\|X_{t(1)}\|/\sqrt{n}\}^{2+c_0} \to 0$, with $0 < c_0 < 1$. $v_{tj} \stackrel{\text{def}}{=} Y_t - a_j - b_j X_{tj}^\top\beta$. Also, exists a constant C_3 such that for all β close to β^* ($\|\beta - \beta^*\| \le C_3$)

$$\|\sum_t\sum_j X_{(0)tj}\omega_{tj}X_{(1)tj}^\top \partial\,\mathsf{E}\,\psi_\tau(v_{tj})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1}).$$

Condition 4. The penalty parameter λ is chosen such that $\lambda = \mathcal{O}(n^{-1/2})$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \scriptstyle{\mathcal{O}}(n^{\alpha_1-\alpha_2/2}\lambda) = \scriptstyle{\mathcal{O}}(n^{-1/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \ne 0\}$ be the true model. Furthermore assume $qh \to 0$ and $h^{-1}\sqrt{q/n} = \scriptstyle{\mathcal{O}}(1)$ as n goes to infinity, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}\{\exp(n^\delta)\}$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$.

Condition 5. The error term ε_t satisfies $\mathsf{Var}(\varepsilon_t) < \infty$. Assume that

$$\sup_t \mathsf{E}\big|\psi_\tau^m(\varepsilon_t)/m!\big| \le s_0 M^m$$
$$\sup_t \mathsf{E}\big|\psi_\tau^m(x_{tj})/m!\big| \le s_0 M^m$$

where s_0 and M are constants, and $\psi_\tau(\cdot)$ is the derivative (a subgradient) of $\rho_\tau(\cdot)$.

Condition 6. The conditional density function $f(\varepsilon|Z_t = u)$ is bounded and absolutely continuous differentiable.

Condition 7. The link function $g(\cdot)$ satisfies a Lipschitz condition in the support of $\beta^\top X$:

$$|g(z) - g(\tilde{z})| \le C|z - \tilde{z}| \tag{25}$$

Conditions 8. $\{X_{tj}, \varepsilon_t\}_{t=-\infty}^{t=\infty}$ are strong mixing process for any j. Moreover, there exists positive constants c_{m1} and c_{m2} such that the $\alpha-$ mixing coefficient for every

$j \in \{1, \cdots, p\}$,

$$\alpha(l) \le \exp(-c_{m1} l^{c_{m2}}), \tag{26}$$

where $c_{m2} > 2\alpha$.

Define $\hat{\beta}^0$ as the minimizer with the loss

$$\tilde{L}_n(\beta) \stackrel{\text{def}}{=} \sum_{j=1}^{n} \sum_{t=1}^{n} \rho_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \beta\big)\omega_{tj}(\beta^*) + n \sum_{l=1}^{p} d_l |\beta_l|,$$

but within the subspace $\{\beta \in \mathbb{R}^p : \beta_{\mathcal{M}_*^c} = \mathbf{0}\}$, and a_j^*, b_j^* are denoted as $g(\beta^{*\top} X)$ and $g'(\beta^{*\top} X)$. The following lemma assures the consistency of $\hat{\beta}^0$,

LEMMA 6.1. *Under Conditions 1-8, recall $d_j = \gamma_\lambda\left(|\beta_j^*|\right)$, we have that*

$$\|\hat{\beta}^0 - \beta^*\| = \mathcal{O}_p\big(\sqrt{q/n} + \|\boldsymbol{d}_{(1)}\|\big) \tag{27}$$

where $\mathbf{d}_{(1)}$ is the subvector of $\boldsymbol{d} = (d_1, \cdots, d_p)^\top$ which contains q elements corresponding to the nonzero $\beta_{(1)}^*$.

PROOF. Note that the last $p - q$ elements of both $\hat{\beta}^0$ and β^* are zero, so it is sufficient to prove $\|\hat{\beta}_{(1)}^0 - \beta_{(1)}^*\| = \mathcal{O}_p\big(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|\big)$.

We first show for $\gamma_n = \mathcal{o}(1)$:

$$\mathrm{P}\left[\inf_{\|\mathbf{u}\|=\mathbf{1}} \left\{\tilde{L}_n(\beta_{(1)}^* + \gamma_n \mathbf{u},\ \mathbf{0}) > \tilde{L}_n(\beta^*)\right\}\right] \to 1.$$

Then there exists a minimizer inside the ball $\{\beta_{(1)} : \|\beta_{(1)} - \beta_{(1)}^*\| \le \gamma_n\}$. Construct $\gamma_n \to 0$ so that for a sufficiently large constant B_0: $\gamma_n > B_0 \cdot \big(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|\big)$. We will show that by the local convexity of $\tilde{L}_n(\beta_{(1)}, \mathbf{0})$ near $\beta_{(1)}^*$, there exists a unique minimizer inside the ball $\{\beta_{(1)} : \|\beta_{(1)} - \beta_{(1)}^*\| \le \gamma_n\}$ with probability tending to 1.

Let $X_{(1)tj}$ denote the subvector of X_{tj} consisting of its first q components.

By the uniform Bahadur representation in Kong et al. (2010), there exist a compact set B with β^* a interior point, such that uniformly over $\mathbf{u} \in B$, we have

$$
\begin{aligned}
&\tilde{L}_n(\beta^*_{(1)} + \gamma_n \mathbf{u}, \mathbf{0}) - \tilde{L}_n(\beta^*_{(1)}, \mathbf{0}) \\
&= -\gamma_n \sum_{t=1}^n \sum_{j=1}^n b^*_j \psi_\tau\big(Y_t - a^*_j - b^*_j X^\top_{(1)tj}\beta^*_{(1)}\big)\omega_{tj}(\beta^*) X^\top_{(1)tj}\mathbf{u} \\
&\quad + \frac{1}{2}\gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b^{*2}_j \partial \mathsf{E}\,\psi_\tau\big(Y_t - a^*_j - b^*_j X^\top_{(1)tj}\beta^*_{(1)} - b^*_j \bar{\gamma}_n X^\top_{(1)tj}\mathbf{u}\big)\omega_{tj}(\beta^*)(X^\top_{(1)tj}\mathbf{u})^2 \\
&\quad + n\sum_{l=1}^q d_l\big(|\beta^*_{(1)l} + \gamma_n u_l| - |\beta^*_{(1)l}|\big) + \mathcal{O}_p(\max\{n\gamma_n^2, n(n/q\log\log n)^{-1}\}) \\
&\stackrel{\text{def}}{=} P_1 + P_2 + P_3 + \mathcal{O}_p(\max\{n\gamma_n^2, n(n/q\log\log n)^{-1}\})
\end{aligned}
$$

where $\bar{\gamma}_n \in [0, \gamma_n]$.

Define $\omega_{tj} \stackrel{\text{def}}{=} \omega_{tj}(\beta^*)$, it is not difficult to derive that $\omega_{tj} = \frac{K_h(Z_{tj})}{n f_Z(Z_j)}\{1 + \mathcal{O}_p(1)\}$ where $Z_t = X_t^\top \beta^*$, $Z_{tj} = Z_t - Z_j$ and $f_Z(\cdot)$ is the density of $Z = X^\top\beta^*$.

For P_1, because $\|\mathbf{u}\| = 1$ and $Y_t = a^*_t + \varepsilon_t$, we get

$$
\begin{aligned}
|P_1| &\le \gamma_n \Big\|\sum_{t=1}^n \sum_{j=1}^n b^*_j \psi_\tau\big(Y_t - a^*_j - b^*_j X^\top_{(1)tj}\beta^*_{(1)}\big)\omega_{tj} X^\top_{(1)tj}\Big\|\{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \Big\|\sum_{j=1}^n b^*_j \Big\{\frac{1}{n}\sum_{t=1}^n \psi_\tau\big(\varepsilon_t + a^*_t - a^*_j - b^*_j Z_{tj}\big)\frac{K_h(Z_{tj})}{f_Z(Z_j)} X_{(1)tj}\Big\}\Big\|\{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \Big\|\sum_{j=1}^n b^*_j\, \mathsf{E}_{\varepsilon_t, X_t}\Big\{\psi_\tau\big(\varepsilon_t + a^*_t - a^*_j - b^*_j Z_{tj}\big)\frac{K_h(Z_{tj})}{f_Z(Z_j)} X_{(1)tj}\Big\}\Big\|\{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \Big\|\sum_{j=1}^n b^*_j\, \mathsf{E}_{Z_t}\Big\{\mathsf{E}_{\varepsilon_t|Z_t}[\psi_\tau\big(\varepsilon_t + a^*_t - a^*_j - b^*_j Z_{tj}\big)]\frac{K_h(Z_{tj})}{f_Z(Z_j)}\mathsf{E}(X_{(1)tj}|Z_t)\Big\}\Big\|\{1 + \mathcal{O}_p(1)\} \\
&= \gamma_n \Big\|\sum_{j=1}^n b^*_j\, \mathsf{E}_{Z_j}[\psi_\tau\big(\varepsilon_j + a^*_j - a^*_j\big)]\{\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j}\}\Big\|\{1 + \mathcal{O}_p(1)\}
\end{aligned}
$$

where $\mathsf{E}_{\varepsilon_t, X_t}$ means taking expectation with respect to the joint distribution of (ε_t, X_t). Furthermore we have

$$
\begin{aligned}
&\mathsf{E}\,\Big\|\sum_{j=1}^n b^*_j\, \mathsf{E}[\psi_\tau\big(\varepsilon_j + a^*_j - a^*_j\big)]\{\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j}\}\Big\| \\
&\le \Big\{\mathsf{E}\,\psi^2_\tau\big(\varepsilon_j\big)\,\mathsf{E}\sum_{j=1}^n b^{*2}_j[\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j}]^\top[\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j}]\Big\}^{1/2} \\
&= \sqrt{n}\{\mathsf{E}\,\psi^2_\tau\big(\varepsilon_j\big)\,\mathrm{tr}(C_{0(1)})\}^{1/2},
\end{aligned}
$$

recall $C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{[g'(Z_j)]^2(\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j})[\mathsf{E}(X_{(1)j}|Z_j) - X_{(1)j}]\}^\top$. We thus arrive at

$$P_1 = \mathcal{O}_p(\gamma_n\sqrt{nq}) \tag{28}$$

because $\operatorname{tr}(C_{0(1)}) = \mathcal{O}(q)$ and $\mathsf{E}\,\psi_\tau^2(\varepsilon_j) < \infty$ by Condition 3.

For P_2, according to the property of kernel estimation, it can be seen that

$$\begin{aligned} P_2 &= \frac{1}{2}\gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b_j^{*2} \partial\, \mathsf{E}_{\varepsilon_t, X_t} \psi_\tau\big(Y_t - a_j^* - b_j^* Z_{tj} - b_j^* \bar{\gamma}_n X_{(1)tj}^\top \mathbf{u}\big) \frac{K_h(Z_{tj})}{n f_Z(Z_j)} (X_{(1)tj}^\top \mathbf{u})^2 \{1 + o_p(1)\} \\ &= \frac{1}{2}\gamma_n^2 \sum_{j=1}^n b_j^{*2} \partial \mathsf{E}\{\psi_\tau\big(Y_t - a_j^* - b_j^* \bar{\gamma}_n X_{(1)tj}^\top \mathbf{u}\big)(X_{(1)tj}^\top \mathbf{u})^2\}\{1 + o_p(1)\} \end{aligned}$$

Let $H_t(c) = \inf_{|v| \le c} \partial\, \mathsf{E}\, \psi(\varepsilon_t - v)$. By lemma 3.1 of Portnoy (1984), we have

$$P_2 \ge \frac{1}{2}\gamma_n^2 \sum_{t=1}^n \sum_{j=1}^n b_j^{*2} H(\gamma_n |X_{(1)tj}^\top \mathbf{u}|)(X_{(1)tj}^\top \mathbf{u})^2 \ge c\gamma_n^2 n \tag{29}$$

for some positive c.

For P_3, it is clear that

$$|P_3| \le n\gamma_n \sum_{l=1}^q d_l |u_l| \le n\gamma_n \|\mathbf{d}_{(\mathbf{1})}\| \tag{30}$$

Combining (28), (29)and (30), the following inequality holds with probability tending to 1 that

$$\tilde{L}_n(\beta_{(1)}^* + \gamma_n \mathbf{u}, \mathbf{0}) - \tilde{L}_n(\beta_{(1)}^*, \mathbf{0}) \ge n\gamma_n\Big(c\gamma_n - \sqrt{q/n} - \|\mathbf{d}_{(1)}\|\Big) \tag{31}$$

$\gamma_n = B_0\big(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|\big)$ and B_0 is a sufficiently large constant, so that the RHS of (31) is larger than 0. Owing to the local convexity of the objective function, there exists a unique minimizer $\hat{\beta}_{(1)}^0$ such that

$$\|\hat{\beta}^0 - \beta^*\| = \|\hat{\beta}_{(1)}^0 - \beta_{(1)}^*\| = \mathcal{O}_p\big(\sqrt{q/n} + \|\mathbf{d}_{(1)}\|\big)$$

Therefore, (27) is proved. □

Recall that $X = (X_{(1)}, X_{(0)})$ and $\mathcal{M}_* = \{1, \ldots, q\}$ is the set of indices at which β are nonzero.

Lemma 1 shows the consistency of $\hat{\beta}^0$, and we need to show further that $\hat{\beta}^0$ is the unique minimizer in $\mathbb{R}^p$ on a set with probability tending to 1.

LEMMA 6.2. *Under conditions 1-8, minimizing the loss function $\tilde{L}_n(\beta)$ has a unique* *global minimizer* $\hat{\beta} = (\hat{\beta}_1^\top, \mathbf{0}^\top)^\top$*, if and only if on a set with probability tending to* 1*,*

$$\sum_{j=1}^{n}\sum_{t=1}^{n}\psi_\tau\big(Y_t - \hat{a}_j - \hat{b}_j X_{tj}^\top \hat{\beta}_\tau\big)\hat{b}_j X_{(1)tj}\omega_{tj}(\beta^*) + n\mathbf{d}_{(1)} \circ \operatorname{sign}(\hat{\beta}_\tau) = 0 \tag{32}$$

$$\|z(\hat{\beta}_\tau)\|_\infty \le n, \tag{33}$$

where

$$z(\hat{\beta}_\tau) \stackrel{\text{def}}{=} \mathbf{d}_{(0)}^{-1} \circ \left\{\sum_{j=1}^{n}\sum_{t=1}^{n} b_j^*\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}_\tau\big)X_{(0)tj}\omega_{tj}(\hat{\beta}_\tau)\right\} \tag{34}$$

where $\circ$ *stands for multiplication element-wise.*

PROOF. According to the definition of $\hat{\beta}_\tau$, it is clear that $\hat{\beta}_{(1)}$ already satisfies condition (32). Therefore we only need to verify condition (33).

To prove (33), a bound for

$$\sum_{t=1}^{n}\sum_{j=1}^{n} b_j^*\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*\big)\omega_{tj}X_{(0)tj} \tag{35}$$

is needed. Define the following kernel function

$$\begin{aligned} & h_d(X_t, a_j^*, b_j^*, Y_t, X_j, a_t^*, b_t^*, Y_j) \\ &= \frac{n}{2}\left\{b_j^*\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top\beta^*\big)\omega_{tj}X_{(0)tj} + b_t^*\psi_\tau\big(Y_j - a_t^* - b_t^* X_{tj}^\top\beta^*\big)\omega_{jt}X_{(0)jt}\right\}_d, \end{aligned}$$

where $\{.\}_d$ denotes the dth element of a vector, $d = 1, \ldots, p-q$.

According to Borisov and Volodko (2009), based on Condition 5:

Define $U_{n,d} \stackrel{\text{def}}{=} \frac{1}{n(n-1)}\sum_{1\le i<j\le n} h_d(X_t, a_j^*, b_j^*, Y_t, X_j, a_i^*, b_t^*, Y_j)$ as the $U-$ statistics for (35). We have, with sufficient large c_{m2} in Condition 8.

$$\mathrm{P}\{|U_{n,d} - \mathsf{E}U_{n,d}| > \varepsilon\} \le c_{m3}\exp(c_{m5}\varepsilon/(c_{m3} + c_{m4}\varepsilon^{1/2}n^{-1/2}))$$

where c_{m3}, c_{m4}, c_{m5} are constants. Moreover, let $\varepsilon = \mathcal{O}(n^{1/2+\alpha})$, as $\alpha < 1/2$, we can further have,

$$\mathrm{P}(\{|U_{n,d} - \mathsf{E}U_{n,d}| > \varepsilon\}) \le c_{m3}\exp(-c_{m6}\varepsilon/2),$$

Define

$$F_{n,d} \stackrel{\text{def}}{=} (n)^{-1}\sum_{t=1}^{n}\sum_{j=1}^{n} b_j\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top\beta^*\big)\omega_{tj}X_{(0)tj},$$

also it is not hard to derive that $U_{n,d} = F_{n,d}n/(n-1)$.
It then follows that

$$\begin{aligned} \mathrm{P}(|F_{n,d} - \mathsf{E}F_{n,d}| > \varepsilon) &= \mathrm{P}(|U_{n,d} - \mathsf{E}U_{n,d}|(n-1)/n > \varepsilon) \\ &\leq 2\exp\big(-Cn^{\alpha+1/2}\big) \end{aligned}$$

Define $\mathcal{A}_n = \{\|F_n - \mathsf{E}F_n\|_\infty \leq \varepsilon\}$, thus

$$\mathrm{P}(\mathcal{A}_n) \geq 1 - \sum_{d=1}^{p-q} \mathrm{P}(|F_{n,d} - \mathsf{E}F_{n,d}| > \varepsilon) \geq 1 - 2(p-q)\exp\big(-Cn^{\alpha+1/2}\big).$$

Finally we get that on the set $\mathcal{A}_n$,

$$\begin{aligned} \|z(\hat{\beta}^0)\|_\infty &\leq \|\mathbf{d}^{-1}_{\mathcal{M}^c_*} \circ F_n\|_\infty + \|\mathbf{d}^{-1}_{\mathcal{M}^c_*} \circ \sum_{t=1}^n \sum_{j=1}^n b_j \big[\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}^0\big) \\ &\quad -\psi_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*\big)\big]\omega_{tj} X_{(0)tj}\|_\infty \\ &\leq \mathcal{O}(n^{1/2+\alpha}/\lambda + \|\mathbf{d}^{-1}_{\mathcal{M}^c_*} \circ \sum_{t=1}^n \sum_{j=1}^n \partial\,\mathsf{E}\,\psi_\tau(v_{tj}) b_j X_{(1)tj}^\top (\hat{\beta}_{(1)} - \beta^*_{(1)})\omega_{tj} X_{(0)tj}\|_\infty), \end{aligned}$$

where v_{tj} is between $Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*$ and $Y_t - a_j^* - b_j^* X_{tj}^\top \hat{\beta}^0$. From Lemma 1,

$$\|\hat{\beta}^0 - \beta^*_{(1)}\|_2 = \mathcal{O}_p\Big(\|\mathbf{d}_{(1)}\| + \sqrt{q}/\sqrt{n}\Big).$$

Choosing $\|\sum_t \sum_j X_{(0)tj}\omega_{tj} X_{(1)tj}^\top \partial\,\mathsf{E}\,\psi_\tau(v_{tj})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1})$, $q = \mathcal{O}(n^{\alpha_2})$, $\lambda = \mathcal{O}(\sqrt{q/n}) = n^{-1/2+\alpha_2/2}$, $0 < \alpha_2 < 1$, $\|\mathbf{d}_{(1)}\| = \mathcal{O}(\sqrt{q}D_n) = \mathcal{O}(n^{\alpha_2/2}D_n)$

$$\begin{aligned} (n)^{-1}\|z(\hat{\beta}^0)\|_\infty &= \mathcal{O}\{n^{-1}\lambda^{-1}(n^{1/2+\alpha} + n^{1-\alpha_1}\sqrt{q}/\sqrt{n} + \|\mathbf{d}_{(1)}\| n^{1-\alpha_1})\} \\ &= \mathcal{O}(n^{-\alpha_2/2+\alpha} + n^{-\alpha_1} + n^{-\alpha_1+\alpha_2/2}D_n/\lambda), \end{aligned}$$

assume $D_n = o(n^{\alpha_1-\alpha_2/2}\lambda)$, and let $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$, with rate $p = \mathcal{O}\{\exp(n^\delta)\}$, then $(n)^{-1}\|z(\hat{\beta}^0)\|_\infty = o_p(1)$. □

Proof of Theorem 1 . The results follows from Lemma 1 and 2. □

Proof of Theorem 2. By Theorem 1, $\hat{\beta}_{\mathbf{w}(1)} = \beta_{(1)}$ almost surely. It then follows from Lemma 2 that

$$\|\hat{\beta}_{\tau(1)} - \beta^*_{(1)}\| = \mathcal{O}_p\{(D_n + n^{-1/2})\sqrt{q}\}.$$

This completes the first part of the theorem.

Given a_j^*, b_j^*, the estimate $\hat{\beta}_\tau$ is:

$$\hat{\beta}_\tau = \arg\min_\beta \sum_{j=1}^n \sum_{t=1}^n \rho_\tau\big(Y_t - a_j^* - b_j^* X_{tj}^\top \beta\big)\omega_{tj} + n\sum_{l=1}^p \gamma_\lambda(|\beta_l^*|)|\beta_l|$$

where $\omega_{tj} = \omega_{tj}(\beta^*)$ and $X_{tj} = X_t - X_j$.
Define $\hat{\xi} \stackrel{\text{def}}{=} \sqrt{n}(\hat{\beta}_\tau - \beta^*)$ and $Y_{tj} = Y_t - a_j^* - b_j^* X_{tj}^\top \beta^*$. As $\omega_{tj}(\tilde{\beta}) = \frac{K_h(Z_{tj})}{nf_Z(Z_j)}\{1 + \mathcal{O}_p(1)\}$, it follows that $\hat{\xi}$ is the minimizer of

$$\begin{aligned} H_n(\xi) &= \sum_{t=1}^n \sum_{j=1}^n \Big\{\rho_\tau\big(Y_{tj} - n^{-\frac{1}{2}} b_j^* X_{tj}^\top \xi\big) - \rho_\tau\big(Y_{tj}\big)\Big\}\frac{K_h(Z_{tj})}{nf_Z(Z_j)}\{1 + \mathcal{O}_p(1)\} \\ &\quad + n\sum_{l=1}^p \gamma_\lambda(|\beta_l^*|)(|\beta_l^* + n^{-1/2}\xi_l| - |\beta_l^*|) \\ &\stackrel{\text{def}}{=} Q_1(\xi)\{1 + \mathcal{O}_p(1)\} + Q_2(\xi), \end{aligned}$$

recall $f_Z(z)$ is the density function of $Z = X_t^\top \beta^*, t = 1, \ldots, n$. We study $Q_1(\xi)$ and $Q_2(\xi)$ respectively.
Let $\Delta_{ij}(\xi) \stackrel{\text{def}}{=} \rho_\tau\{Y_{tj} - n^{-\frac{1}{2}} g'(Z_j) X_{tj}^\top \xi\} - \rho_\tau\big(Y_{tj}\big) - n^{-\frac{1}{2}}\psi_\tau(Y_{tj})g'(Z_j)X_{tj}^\top \xi$. It can be seen that

$$\begin{aligned} Q_1(\xi) &= \Big\{\sum_{t=1}^n \sum_{j=1}^n n^{-1/2}\psi_\tau(Y_{tj})g'(Z_j)\frac{K_h(Z_{tj})}{nf_Z(Z_j)} X_{tj}^\top \xi \\ &\quad + \sum_{t=1}^n \sum_{j=1}^n \Delta_{ij}(\xi)\frac{K_h(Z_{tj})}{nf_Z(Z_j)}\Big\}\{1 + \mathcal{O}_p(1)\} \\ &\stackrel{\text{def}}{=} [A^\top \xi + I(\xi)]\{1 + \mathcal{O}_p(1)\} \end{aligned}$$

Recall that $Y_{tj} = \varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj} + o_p(1)$.
Therefore we have

$$\begin{aligned} A &= \sum_{t=1}^n \sum_{j=1}^n \frac{1}{\sqrt{n}}\psi_\tau(\varepsilon_t + a_t^* - a_j^* - b_j^* Z_{tj})g'(Z_j)\frac{K_h(Z_{tj})}{nf_Z(Z_j)} X_{tj}\{1 + \mathcal{O}_p(1)\} \\ &= \sum_{t=1}^n \frac{1}{\sqrt{n}}\psi_\tau(\varepsilon_t)g'(Z_t)\{\mathsf{E}(X|Z_t) - X_t\}\{1 + \mathcal{O}_p(1)\}. \end{aligned}$$

Similarly we have, as before via the uniform Bahadur representation.

$$
\begin{aligned}
I(\xi) &= \sum_{t=1}^{n}\sum_{j=1}^{n} \Delta_{ij}(\xi)\frac{K_h(Z_{tj})}{nf_Z(Z_j)} \\
&= \sum_{t=1}^{n}\sum_{j=1}^{n} \big\{\rho_\tau\big(Y_{tj} - n^{-\frac{1}{2}} g'(Z_j)X_{tj}^\top\xi\big) - \rho_\tau\big(Y_{tj}\big) \\
&\quad - n^{-\frac{1}{2}}\psi_\tau(Y_{tj})g'(Z_j)X_{tj}^\top\xi\big\}\frac{K_h(Z_{tj})}{nf_Z(Z_j)} \\
&= \sum_{j=1}^{n} \mathsf{E}_{\varepsilon_t,X_t}\Big[\big\{\rho_\tau\big(Y_{tj} - n^{-\frac{1}{2}} g'(Z_j)X_{tj}^\top\xi\big) - \rho_\tau\big(Y_{tj}\big) \\
&\quad - n^{-\frac{1}{2}}\psi_\tau(\varepsilon_t)g'(Z_t)X_{tj}^\top\xi\big\}\frac{K_h(Z_{tj})}{f_Z(Z_j)}\Big]\{1+\mathcal{O}(1)\} \\
&= \sum_{j=1}^{n} \mathsf{E}_{Z_t}\mathsf{E}_{\varepsilon_t,X_t|Z_t}\Big[\big\{\rho_\tau\big(Y_{tj} - n^{-\frac{1}{2}} g'(Z_j)X_{tj}^\top\xi\big) - \rho_\tau\big(Y_{tj}\big) \\
&\quad - n^{-\frac{1}{2}}\psi_\tau(\varepsilon_t)g'(Z_t)X_{tj}^\top\xi\big\}\frac{K_h(Z_{tj})}{f_Z(Z_j)}\Big]\{1+\mathcal{O}(1)\} \\
&= \sum_{j=1}^{n} \Big\{\mathsf{E}_{\varepsilon,Z}\rho_\tau\big(\varepsilon - n^{-\frac{1}{2}} g'(Z_j)(X_j - \mathsf{E}(X_j|Z_j))^\top\xi\big) - \mathsf{E}_{\varepsilon}\rho_\tau\big(\varepsilon\big) \\
&\quad - n^{-\frac{1}{2}}\mathsf{E}_{\varepsilon,Z}[\psi_\tau(\varepsilon)]g'(Z_j)(X_j - \mathsf{E}(X_j|Z_j))^\top\xi\Big\}\{1+\mathcal{O}(1)\}
\end{aligned}
$$

$$
\begin{aligned}
&= (2n)^{-1}\sum_{j=1}^{n}\mathsf{E}\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_j)\}[g'(Z_j)]^2\xi^\top\{X_j - \mathsf{E}(X_j|Z_j)\}\{X_j - \mathsf{E}(X_j|Z_j)\}^\top\xi\{1+\mathcal{O}(1)\} \\
&= \frac{1}{2}\xi^\top\mathsf{E}\{\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_j)\}[g'(Z_j)]^2\{X_j - \mathsf{E}(X_j|Z_j)\}\{X_j - \mathsf{E}(X_j|Z_j)\}^\top\}\xi\{1+\mathcal{O}(1)\} \\
&\stackrel{\text{def}}{=} \frac{1}{2}\xi^\top C_0\xi\{1+\mathcal{O}(1)\},
\end{aligned}
$$

where $C_0 \stackrel{\text{def}}{=} \mathsf{E}\{\partial\,\mathsf{E}\,\psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_i|Z_t) - X_i)(\mathsf{E}(X_i|Z_t) - X_i)\}^\top$.

About $Q_2(\xi)$, we find that if $\beta_l^* = 0$, that is $q < l \le p$, $\sqrt{n}(|\beta_l^* + n^{-1/2}\xi| - |\beta_l^*|) = |\xi|$, otherwise when $1 \le l \le q$, $\sqrt{n}(|\beta_l^* + n^{-1/2}\xi| - |\beta_l^*|) \to \xi\mathrm{sign}(\beta_l^*)$. In addition, Theorem 1 implies that $\hat{\beta}_{\mathbf{w}(2)}$ will converge to zero with probability one. Thus

$$
\begin{aligned}
Q_2(\xi) &= n\sum_{l=1}^{p}\gamma_\lambda(|\beta_l^*|)(|\beta_l^* + n^{-1/2}\xi_l| - |\beta_l^*|) \\
&= n\sum_{l=1}^{q}\gamma_\lambda(|\beta_l^*|)\xi_l\mathrm{sign}(\beta_l^*)\{1+\mathcal{O}_p(1)\}.
\end{aligned}
$$

As $\gamma_\lambda(|\beta_l^*|) = \mathcal{O}(D_n) = \scriptstyle\mathcal{O}(\sqrt{n})$, the loss function satisfies

$$H_n(\xi) = D_n(\xi)\{1 + o(1)\} \stackrel{\text{def}}{=} [-\xi_{(1)}^\top A_{(1)} + \frac{1}{2}\xi_{(1)}^\top C_{0(1)}\xi_{(1)}]\{1 + o(1)\}$$

where $A_{(1)}$ is the sub-vector of A consisting of its first q components, and $C_{0(1)}$ is the up-left $q \times q$ sub-matrix of C_0. Applying the uniform Bahadur representation, we obtain $\hat{\xi}_{(1)} = \sqrt{n}(\hat{\beta}_{(1)} - \beta_{(1)}^*) = C_{0(1)}^{-1} A_{(1)} + o_p(1)$.

Recall that $A_{(1)} = \sum_{t=1}^{n} \frac{1}{\sqrt{n}} \psi_\tau(\varepsilon_t) g'(Z_t)\{\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}\}\{1 + o_p(1)\}$, and $\mathbf{b}$ is a unit vector in $\mathbf{R}^q$. Thus we have

$$\begin{aligned}
\sqrt{n}\mathbf{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} (\hat{\beta}_{(1)} - \beta_{(1)}^*) &= \mathbf{b}^\top C_{0(1)}^{1/2} C_{1(1)}^{-1/2} C_{0(1)}^{1/2} C_{0(1)}^{-1} \\
&\quad \times \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \psi_\tau(\varepsilon_t) g'(Z_t)\{\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}\}\{1 + o_p(1)\} \\
&\xrightarrow{\mathcal{L}} \mathbb{N}(0, 1)
\end{aligned}$$

where $C_{1(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\mathsf{E}\{\psi_\tau^2(\varepsilon_t)|Z_t\}[g'(Z_t)]^2[\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}][\mathsf{E}(X_{(1)}|Z_t) - X_{t(1)}]^\top\}$, and $C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{\partial \mathsf{E}\, \psi_\tau(\varepsilon_t)|Z_t\}\{[g'(Z_t)]^2(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})(\mathsf{E}(X_{t(1)}|Z_t) - X_{t(1)})\}^\top$. The asymptotic normality of $\hat{\beta}_{(1)}$ is therefore proved. $\square$

Proof of Theorem 3. We note that

$$\sqrt{nh}\left\{\hat{g}(x^\top\hat{\beta}_\tau) - g(x^\top\beta^*)\right\} = \sqrt{nh}\left\{\hat{g}(x^\top\hat{\beta}_\tau) - \hat{g}(x^\top\beta^*)\right\} + \sqrt{nh}\left\{\hat{g}(x^\top\beta^*) - g(x^\top\beta^*)\right\}$$

As ρ_τ is strictly convex, then $\hat{g}(\cdot)$ is continuous almost surely. As $qh \to 0$, the consistency of $\hat{\beta}$ in Theorem 2 implies $\sqrt{nh}\left\{\hat{g}(x^\top\hat{\beta}_\tau) - \hat{g}(x^\top\beta^*)\right\} = o_p(1)$. Consequently it is sufficient to prove

$$\sqrt{nh^3}\sqrt{\{f_Z(z)\mu_2^2\}/(\nu_2\sigma_\tau^2)}\,\{\hat{g}(x^\top\beta^*) - g(x^\top\beta^*)\} \xrightarrow{\mathcal{L}} \mathbb{N}(0,\ 1),$$

recall μ_2 and ν_2 are defined as $\mu_j \stackrel{\text{def}}{=} \int u^j K(u)du$ and $\nu_j \stackrel{\text{def}}{=} \int u^j K^2(u)du$.

We now prove equation (36). Let $Z_t = X_t^\top\beta^*$ and $z = x^\top\beta^*$. Recall that $\hat{g}(z) = \hat{a}$ and $\hat{g}'(z) = \hat{b}$ where

$$(\hat{a}, \hat{b}) \stackrel{\text{def}}{=} \underset{(a,b)}{\arg\min} \sum_{t=1}^{n} \rho_\tau\{Y_t - a - b(Z_t - u)\}\omega_t(\beta^*) \tag{36}$$

where $\omega_t(\beta^*) = K_h(Z_t - z)/\sum_{k=1}^{n} K_h(Z_k - z) = \{1 + o_p(1)\}K_h(Z_t - z_t)/\{nf_Z(z)\}$

Denote $\theta = [\sqrt{nh}\{a - g(z)\},\ \sqrt{nh^3}\{b - g'(z)\}]^\top]^\top$ and $\hat{\theta} = [\sqrt{nh}\{\hat{g}(z) - g(z)\},$

$\sqrt{nh^3}\{\hat{g}'(z) - g'(z)\}]^\top$. Also let $\mathbf{Z}_t \stackrel{\text{def}}{=} (1, (Z_t - z)/h)^\top$ and $\delta_t \stackrel{\text{def}}{=} g(Z_t) - g(z) - g(z)(Z_t - z)$. Then $\hat{\theta}$ is the minimizer of

$$L_{g,n}(\theta) \stackrel{\text{def}}{=} \sum_{t=1}^{n} \{\rho_\tau\big(\varepsilon_t + \delta_t - (nh)^{-1/2}\mathbf{Z}_t^\top \theta\big) - \rho_\tau\big(\varepsilon_t + \delta_t\big)\} \frac{K_h(Z_t - z)}{f_Z(z)}\{1 + \mathcal{O}(1)\}. \quad (37)$$

By similar expansion of $Q_1(\xi)$ in the proof Theorem 2, we have

$$\begin{aligned}
\sqrt{nh} L_{g,n}(\theta) &= \frac{1}{\sqrt{nh}} \sum_{t=1}^{n} \psi_\tau\big(\varepsilon_t + \delta_t\big)\mathbf{Z}_t^\top \theta \frac{K\{(Z_t - z)/h\}}{f_Z(z)}\{1 + \mathcal{O}(1)\} \\
&\quad + \frac{1}{2nh} \sum_{t=1}^{n} \{\partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t + \delta_t\big)\}(\mathbf{Z}_t^\top \theta)^2 \frac{K\{(Z_t - z)/h\}}{f_Z(z)}\{1 + \mathcal{O}(1)\} \\
&= A_n^\top \theta + \frac{1}{2}\theta^\top S_n \theta\{1 + \mathcal{O}(1)\}
\end{aligned}$$

where

$$\begin{aligned}
A_n &= \frac{1}{\sqrt{nh}} \sum_{t=1}^{n} \psi_\tau\big(\varepsilon_t + \delta_t\big)\mathbf{Z}_t \frac{K\{(Z_t - z)/h\}}{f_Z(z)} \\
S_n &= \frac{1}{nh} \sum_{t=1}^{n} \{\partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t + \delta_t\big)\}\mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)}.
\end{aligned}$$

The leading term is then

$$\begin{aligned}
S_n &\stackrel{\text{def}}{=} \frac{1}{h}\,\mathsf{E}\left\{\partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t + \delta_t\big)\mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)} | Z_t]\right\} \\
&= \frac{1}{h}\,\mathsf{E}\left\{\partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t + \delta_t\big)|Z_t]\mathbf{Z}_t \mathbf{Z}_t^\top \frac{K\{(Z_t - z)/h\}}{f_Z(z)}\right\} \\
&= \partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t\big) \int \begin{pmatrix} 1 & (\xi - u)/h \\ (\xi - u)/h & (\xi - z)^2/h^2 \end{pmatrix} K\left(\frac{\xi - z}{h}\right)\frac{d\xi}{h} \\
&= [\partial \,\mathsf{E}\, \psi_\tau\big(\varepsilon_t\big)] \begin{pmatrix} 1 & 0 \\ 0 & \mu_2 \end{pmatrix} \{1 + \mathcal{O}(1)\} \stackrel{\text{def}}{=} S\{1 + \mathcal{O}(1)\}
\end{aligned}$$

where $\mu_2 = \int t^2 K(t)\, dt$. Therefore it follows that $\hat{\theta} = -S^{-1}A_n\{1 + \mathcal{O}_p(1)\}$.

It is clear that $\Sigma^{-1/2}A_n$ is asymptotically distributed as a bivariate normal distribution

$\mathbb{N}(0,1)$. The asymptotic bias $m_A = \mathcal{O}(1)$, in particular

$$
\begin{aligned}
m_A &= \frac{1}{\sqrt{h}} \mathsf{E}\left\{\psi_\tau\big(\varepsilon_t+\delta_t\big)\mathbf{Z}_t \frac{K\{(Z_t-z)/h\}}{f_Z(z)}\right\} \\
&= \frac{1}{\sqrt{h}} \mathsf{E}\left\{\mathsf{E}\left[\psi_\tau\big(\varepsilon_t+\delta_t\big)|Z_t\right]\mathbf{Z}_t \frac{K\{(Z_t-z)/h\}}{f_Z(z)}\right\} \\
&= \frac{1}{\sqrt{h}} \iint \psi_\tau\big\{\varepsilon+g(\xi)-g(z)-g'(z)(\xi-z)\big\} f_E(\varepsilon)d\varepsilon \\
&\quad \times \begin{pmatrix} 1 \\ (\xi-z)/h \end{pmatrix} K((\xi-z)/h)d\xi \\
&= \frac{1}{2} h^{5/2} g''(z)\mu_2[\partial\,\mathsf{E}\,\psi_\tau(\varepsilon)] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \{1+\mathcal{O}(1)\}.
\end{aligned}
$$

The scaling variance covariance matrix is then

$$
\begin{aligned}
\Sigma &= \frac{1}{h}\mathsf{E}\left\{\psi_\tau^2\big(\varepsilon_t+\delta_t\big)\mathbf{Z}_t\mathbf{Z}_t^\top\Big[\frac{K\{(Z_t-z)/h\}}{f_Z(z)}\Big]^2\right\}\{1+\mathcal{O}(1)\} \\
&= \frac{1}{h}\int\int \psi_\tau^2\big\{\varepsilon+g(\xi)-g(z)-g'(z)(\xi-z)\big\} f_E(\varepsilon)d\varepsilon \\
&\quad \times \begin{pmatrix} 1 & (\xi-z)/h \\ (\xi-z)/h & (\xi-z)^2/h^2 \end{pmatrix} \frac{K^2\{(Z_t-z)/h\}}{f_Z(z)} d\xi\{1+o(1)\} \\
&= \frac{1}{f_Z(z)} \mathsf{E}[\psi_\tau^2(\varepsilon)] \begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2 \end{pmatrix} \{1+\mathcal{O}(1)\},
\end{aligned}
$$

where $\nu_k = \int t^k K^2(t)dt$.

Thus we finally obtain that as n tends to infinity, $\Sigma^{-1/2}S\hat{\theta} - \Sigma^{-1/2}Sm_A$ converges in distribution to $\mathbb{N}(0,1)$. Slight algebra gives

$$
\begin{aligned}
\Sigma^{-1/2}Sm_A &= \frac{1}{2}h^{5/2}g''(z)f_Z(z)^{1/2}\nu_0^{-1/2}\{\partial\,\mathsf{E}\,\psi_\tau\big(\varepsilon\big)\}^2\{\mathsf{E}\,\psi_\tau^2\big(\varepsilon\big)\}^{-1/2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
S^{-1}\Sigma S^{-1} &= \frac{\mathsf{E}[\psi_\tau^2(\varepsilon)]}{\{[\partial\,\mathsf{E}\,\psi_\tau\big(\varepsilon\big)]\}^2}\frac{1}{f_Z(z)}\begin{pmatrix} 1 & 0 \\ 0 & \mu_2^{-1} \end{pmatrix}\begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \mu_2^{-1} \end{pmatrix} \\
&= \sigma_{\mathbf{w}}^2\frac{1}{f_Z(z)}\begin{pmatrix} \nu_0 & 0 \\ 0 & \nu_2/\mu_2^2 \end{pmatrix}.
\end{aligned}
$$

This completes the proof. $\square$

7. Appendix B: Tables and Figures

	Depositories (100)				
ASBC	Associated Banc-Corp	CBSH	Commerce Bancshares, Inc.	HBAN	Huntington Bancshares
AF	Astoria Financial Corporation	CBU	Community Bank System, Inc.	IBKC	IberiaBank Corp
BANF	BancFirst Corporation	CACC	Credit Acceptance Corp.	INDB	Independent Bank Corp.
BXS	BancorpSouth, Inc.	CFR	Cullen-Frost Bankers, Inc.	IBOC	International Bancshares Corp.
BAC	Bank of America	CVBF	CVB Financial Corp.	JPM	JP Morgan Chase & Co
BOH	Bank of Hawaii Corporation	DLLR	DFG Gloval Corp.	KRNY	Kearny Financial Corp.
BK	Bank of New York Mellon	EWBC	East West Bancorp, Inc.	KEY	Keycorp
OZRK	Bank of the Ozarks, Inc.	ECPG	Encore Capital Group, Inc.	MTB	M & T Bank
BANR	Banner Corporation	EEFT	Euronet Wolrdwide, Inc.	MBFI	MB Financial Inc.
BBT	BB & T	FNB	F.N.B. Corporation	NPBC	National Penn Bancshares Inc.
BBCN	BBCN Bancorp Inc.	FCNCA	First Citizens Bancshares Inc.	NBTB	NBT Bancorp, Inc.
BOKF	BOK Financial Corporation	FCF	First Commonwealth Financial Corp.	NNI	Nelnet, Inc.
BPFH	Boston Private Fin. Holdings, Inc.	FFBC	First Financial Bancrop.	NTRS	Northern Trust Corp.
BRKL	Brookline Bancorp, Inc.	FFIN	First Financial Bancshares Inc.	NWBI	Northwest Bancshares, Inc.
CSE	CapitalSource Inc.	FHN	First Horizon National Corporation	OCN	Ocwen Financial Corp.
CFFN	Capitol Federal Financial, Inc.	FMBI	First Midwest Bancorp Inc.	ONB	Old National Bancorp.
CATY	Cathay General Bancorp	FNFG	First Niagara Financial Group, Inc.	PCBC	Pacific Capital Corp.
CPF	Central Pacific Corp.	FMER	FirstMerit Corporation	PACW	PacWest Bancorp
CHFC	Chemical Financial Corporation	FBC	Flagstar Bancorp Inc.	PRK	Park National Corp.
C	Citigroup	FULT	Fulton Financial Corporation	PBCT	Peoples United Financial
CRBC	Citizens Republic Bancorp, Inc.	GBCI	Glasier Bancorp Inc.	PNFP	Pinnacle Fin. Partners, Inc.
CYN	City National Corp.	HBHC	Hancock Holding Company	PNC	PNC Financial Services
COLB	Columbia Banking System, Inc.	HTH	Hilltop Holdings Inc.	BPOP	Popular, Inc.
CMA	Comerica Inc.	HCBK	Hudson City Bancorp	PVTB	Privatebancorp, Inc.
PB	Prosperity Bancshares	ZION	Zion	KMPR	Kemper Corporation
PFS	Provident Financial Services, Inc.		**Insurance (56)**	LNC	Lincoln National Corporation
RF	Regions Financial	AFL	Aflac	L	Loews Corporation

SRCE	1st Source Corporation	Y	Alleghany Corp	MKL	Markel Corp.
STBA	S & T Bancorp, Inc.	ALL	Allstate Corp	MMC	Marsh & McLennan
SCBT	SCBT Financial Corporation	AEL	American Equity Inv Life Hldg Co.	MBI	MBIA Inc.
SBNY	Signature Bank	AFG	American Financial Group Inc	MCY	Mercury General Corporation
STT	State Street	AIG	American International Group, Inc.	MET	MetLife Inc
STI	SunTrust Banks, Inc.	ANAT	American National Insurance Co.	NFP	National Financial Partners Corp.
SUSQ	Susquehanna Bancshares, Inc.	AON	AON Corp	NWLI	National Western Life Ins Company
SIVB	SVB Financial Group	AJG	Arthur J Callagher & Co.	NAVG	Navigators Group, Inc.
SNV	Synovus Financial Corporation	AIZ	Assurant	ORI	Old Republic International Corp.
TCB	TCF Financial Corporation	BHLB	Berkshire Hills Bancorp Inc.	PFG	Principal Financial Group
TCBI	Texas Capital Bancshares Inc.	BRO	Brown & Brown Inc.	PRA	ProAssurance Corporation
TMP	Tompkins Financial Corporation	CB	Chubb Corp	PGR	Progressive
UMBF	UMB Financial Corporation	CINF	Cincinatti Financial Corp	PL	Protective Life Corporation
UMPQ	Umpqua Holdings Corporation	CIA	Citizens, Inc.	PRU	Prudential Financial, Inc.
UBSI	United Bancshares, Inc.	CNA	CNA Financial Corporation	RDN	Radian Group Inc.
USB	US Bancorp	NO	CNO Financial Group, Inc.	RLI	RLI Corp.
VLY	Valley National Bancorp	ERIE	Erie Indemnity Company	SAFT	Safety Insurance Group, Inc.
WAFD	Washington Federal Inc.	FFG	FBL Financial Group Inc.	SIGI	Selective Insurance Group Inc.
WBS	Webster Financial Corp.	FNF	Fidelity National Financial	SFG	StanCorp Financial Group Inc.
WFC	Wells Fargo & Co	HIG	Hartford Fin. Services Group, Inc.	STFC	State Auto Financial Corporation
WSBC	WesBanco, Inc.	HCC	HCC Insurance Holdings Inc.	THG	The Hannover Insurance Group
WABC	Westamerica Bancorp.	HCSG	Healthcare Services Group Inc.	TRV	The Travelers Companies, Inc.
WTFC	Wintrust Financial Corporation	HMN	Horacle Mann Educators Corp.	TMK	Torchmark
WRLD	World Acceptance Corp.	IPCC	Infinity Property and Casualty Corp.	TWGP	Tower Group, Inc.
UFCS	United Fire Group, Inc.	SF	Stifel Financial Corporation	SLM	SLM Corporation
UNM	Unum Group	TROW	T. Rowe Price Group, Inc.	AMTD	TD Ameritrade
WRB	W. R. Berkley Corporation	SCHW	The Charles Schwab Corporation		
WTM	White Mountains Ins Group, Ltd.	TRMK	Trustmark Corporation		
	Broker-Dealers (25)	WDR	Waddell & Reed Financial Inc.		
AMG	Affiliated Managers Group Inc.	WETF	WisdomTree Investments, Inc		
AB	AllianceBernstein Holding L.P.		**Others (19)**		

BGCP	BGC Partners, Inc.	ABM	ABM Industries, Inc.
BLW	BlackRock Lim. Duration Inc. Trust	AXP	American Express
EPHC	Epoch Investment Partners, Inc.	AMP	Ameriprise Financial
ETFC	E-Trade Financial	BLK	BlackRock, Inc.
FGG	Fairfield Greenwich group	COF	Capital One Financial
FII	Federal Investors, Inc.	CME	CME Group.
GBL	GAMCO Investors, Inc.	DNB	Dun & Bradstreet Corp.
GS	Goldman Sachs	EV	Eaton Vance
IVZ	Invesco Ltd.	EFX	Equifax Inc.
JEF	Jefferies Group, Inc.	FITB	Fifth Third Bancorp
KFN	KKR Financial Holdings LLC	BEN	Franklin Resources
MKTX	Marketaxess Holdings	JNS	Janus Capital
MS	Morgan Stanley	LM	Legg Mason
MORN	Morningstar Inc.	MCO	Moody's Corp.
NDAQ	Nasdaq OMX Group Inc.	PRAA	Portfolio Recovery Associates Inc.
RJF	Raymond James Financial Inc.	ROL	Rollins Inc.
STSA	Sterling Financial Corporation	SEIC	SEI Investments Company

Table 2: Financial companies with tickers classified by industry: depositories (100), insurance (56), broker-dealers (25) and others (19).

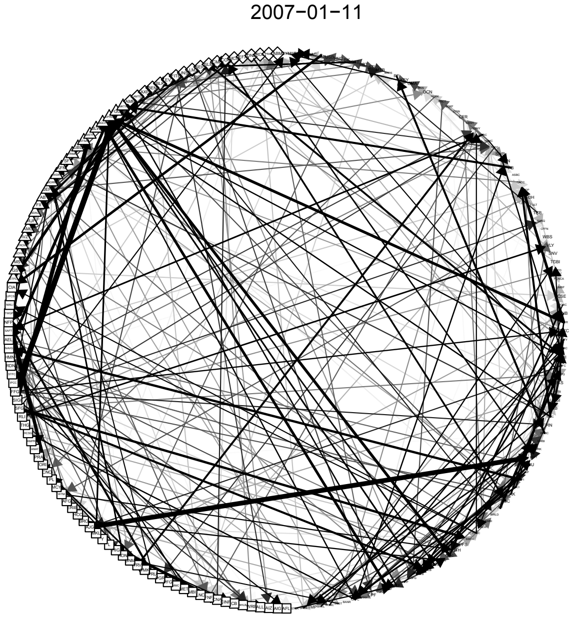

Figure 1: A risk network formed by 200 financial institutions without thresholding of coefficients, a circular representation of a weighted adjacency matrix, Depositories: a circle, Insurance: a square, Broker-Dealers: a triangle, Others: a diamond, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

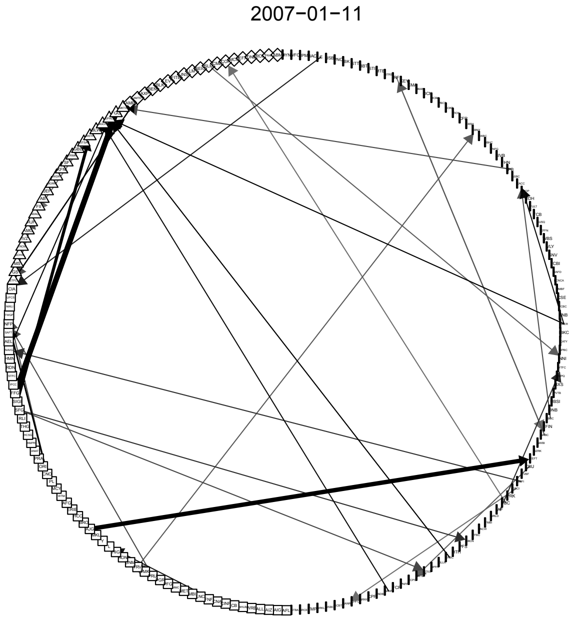

Figure 2: A risk network formed by 200 financial institutions with thresholding of coefficients (all links below the average value areset to 0); a circular representation of a weighted adjacency matrix, Depositories: a circle, Insurance: a square, Broker-Dealers: a triangle, Others: a diamond, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

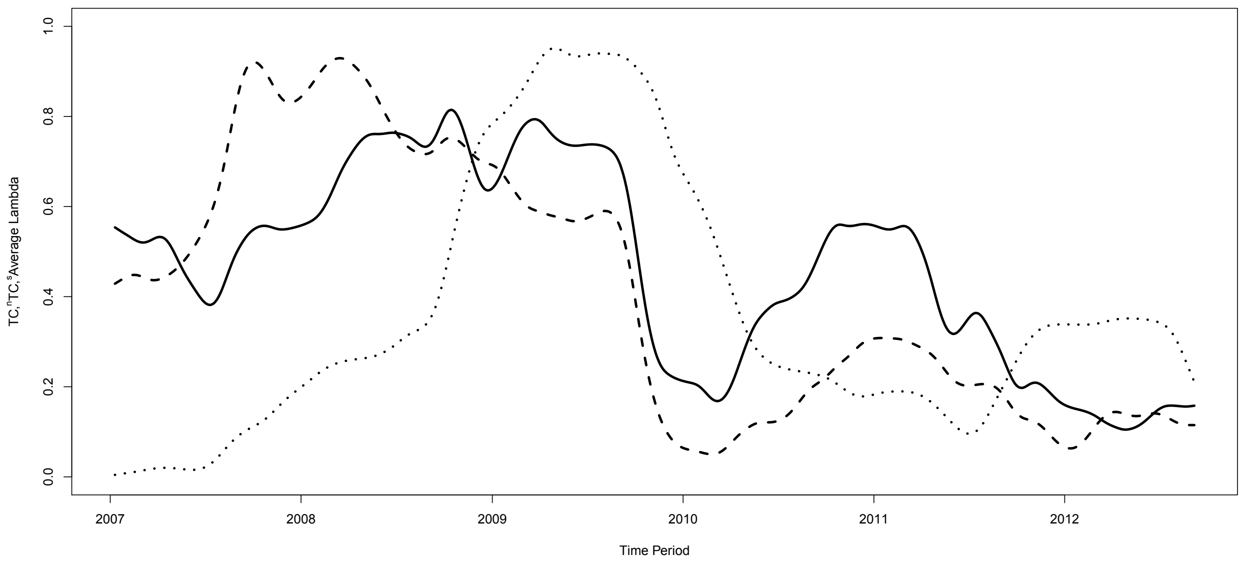

Figure 3: Total Connectedness standardized on $[0, 1]$ scale, TC^s: solid line, TC^s: dashed line, $\bar{\lambda}$: dotted line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

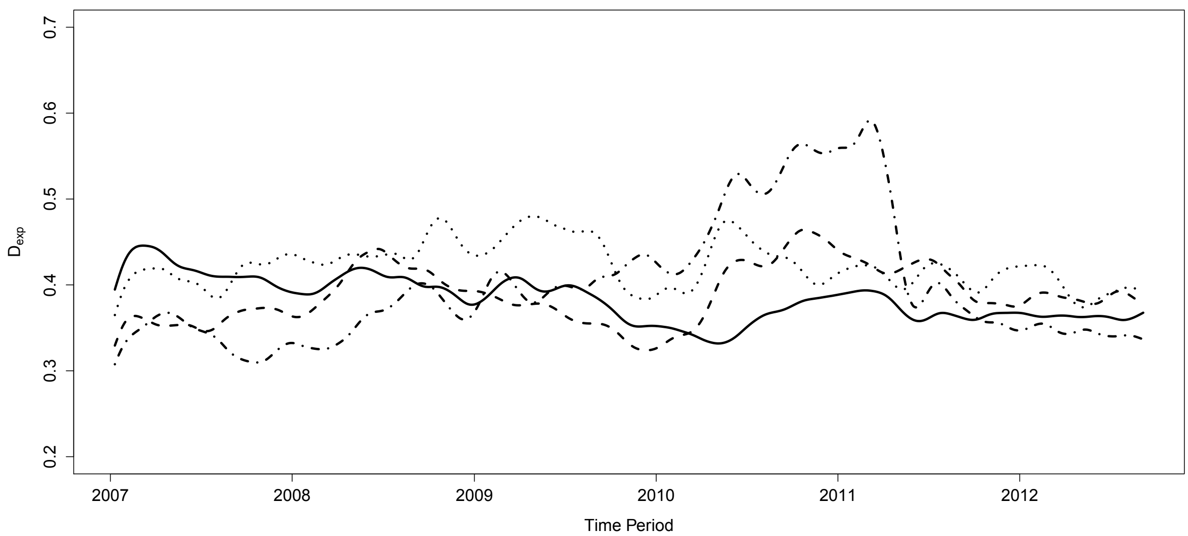

Figure 4: D^m_{exp}, Depositories: solid line, Insurance: dashed line, Broker-Dealers: dotted line, Others: dash-dot line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

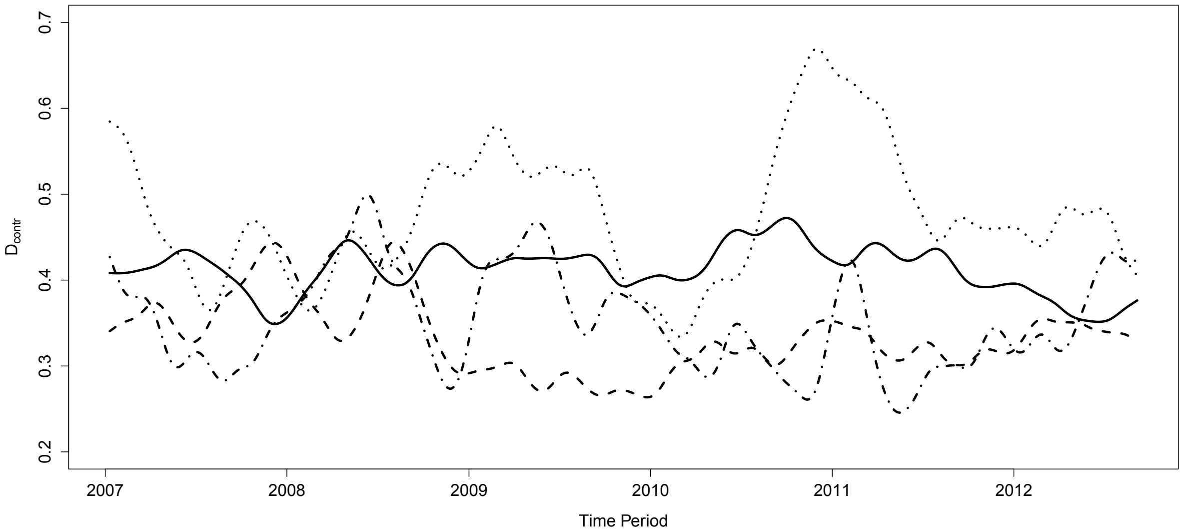

Figure 5: D^m_{contr}, Depositories: solid line, Insurance: dashed line, Broker-Dealers: dotted line, Others: dash-dot line, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

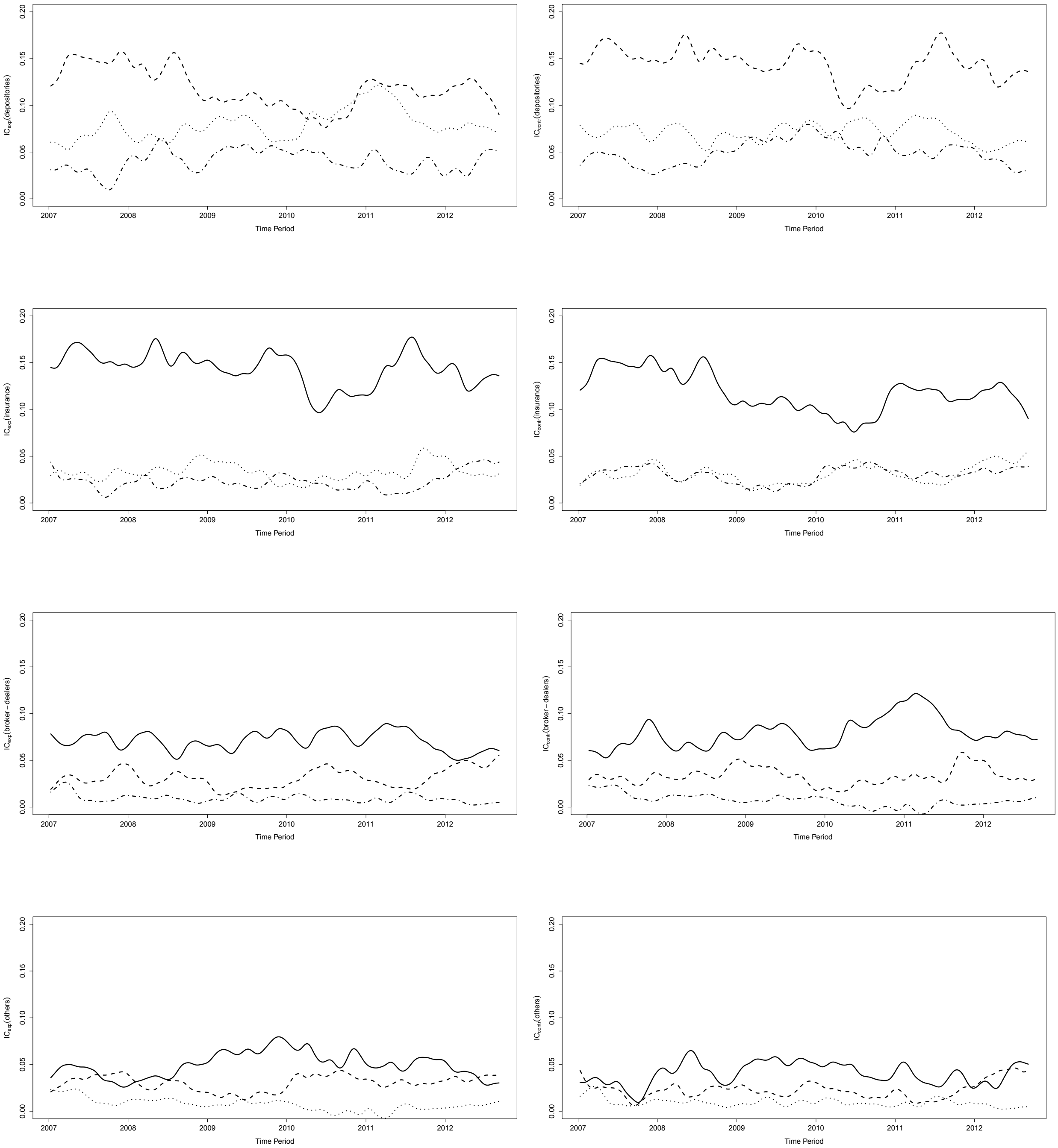

Figure 6: Connectivity between Sectors: IC_{exp} (left) and IC_{contr} (right); Depositories (row 1): solid line, Insurance (row 2): dashed line, Broker-Dealers (row 3): dotted line, Others (row 4): dash-dot line; $T = 1669$, $\tau = 0.05$, window size $n = 125$.

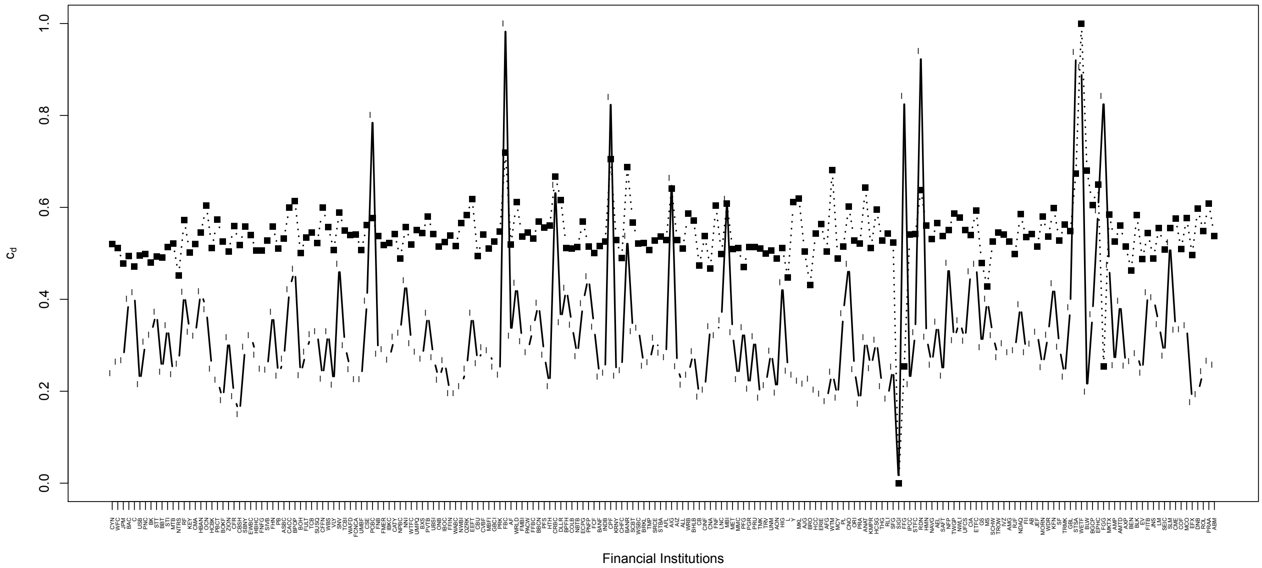

Figure 7: Indegree of 200 financial institutions, $c^{m,k}_{d^-} = \sum_{i=1}^{p} |\widehat{\beta}_{s_{m,k}|-i}|$ (sum over all windows): solid line, circles; $c^{m,k}_{d_n^-} = \sum_{i=1}^{p} d_{s_{m,k}|-i}$ (sum over all windows): dotted line, squares; standardized on $[0, 1]$ scale, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

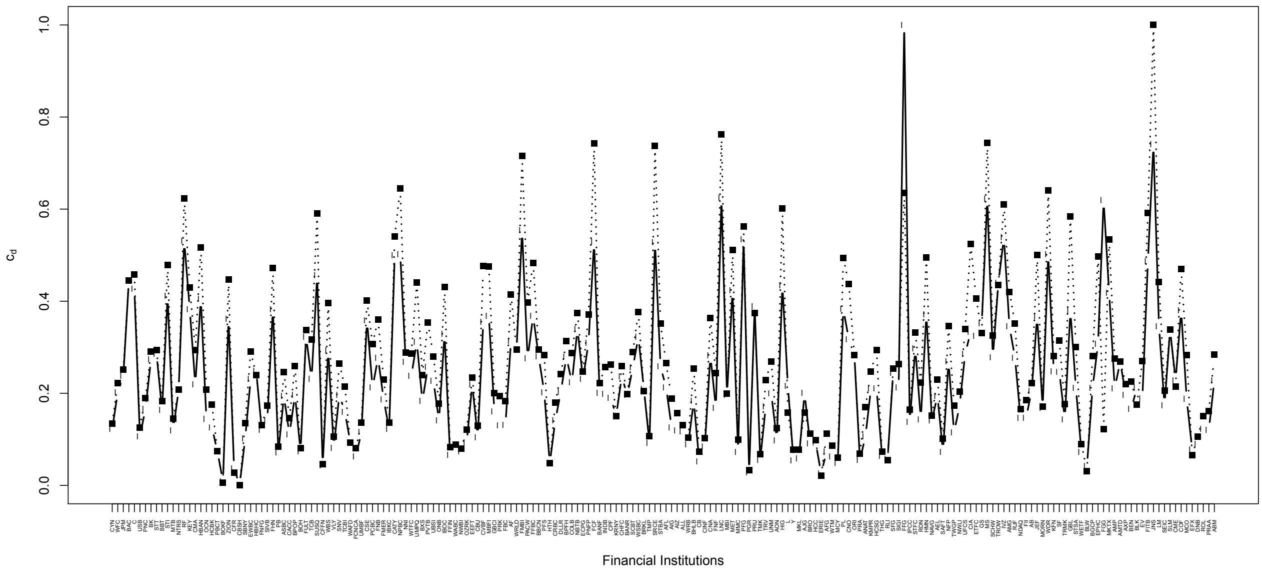

Figure 8: Outdegree of 200 financial institutions, $c^{m,k}_{d^+} = \sum_{j=1}^{p} |\widehat{\beta}_{-j|s_{m,k}}|$ (sum over all windows): solid line, circles; $c^{m,k}_{d_n^+} = \sum_{j=1}^{p} d_{-j|s_{m,k}}$ (sum over all windows): dotted line, squares; standardized on $[0, 1]$ scale, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

	Ticker	Ratio IN	Total Sum of IN Coefficients	Total Number of IN Links
180	FGG	0.99	1416.99	1417
144	FFG	0.99	1416.99	1417
147	RDN	0.28	1021.09	3562
159	MS	0.28	671.50	2390
87	PNFP	0.25	733.87	2875
122	HIG	0.25	728.37	2855
48	PCBC	0.24	786.25	3223
189	JNS	0.24	657.57	2730
192	SLM	0.23	733.99	3097
152	NFP	0.23	728.83	3076

Table 3: Top 10 financial institutions ranked according to the ratio of total sum of IN coefficients to total sum of number of IN links.

	Ticker	Ratio IN	Total Sum of IN Coefficients	Total Number of IN Links
176	WETF	0.01	77.25	5584
72	FBC	0.18	726.54	4014
91	CPF	0.10	425.74	3935
94	BANR	0.09	349.72	3843
131	WTM	0.06	242.98	3801
177	BLW	0.04	187.27	3797
175	STSA	0.13	494.79	3763
81	CRBC	0.16	599.66	3727
179	EPHC	0.10	386.51	3628
137	ANAT	0.08	290.53	3590

Table 4: Top 10 financial institutions ranked according to the total sum of number of IN links.

	Ticker	Ratio IN	Total Sum of IN Coefficients	Total Number of IN Links
180	FGG	0.99	1416.99	1417
144	FFG	0.99	1416.99	1417
147	RDN	0.28	1021.09	3562
48	PCBC	0.24	786.25	3223
192	SLM	0.23	733.99	3097
87	PNFP	0.25	733.87	2875
152	NFP	0.23	728.83	3076
122	HIG	0.25	728.37	2855
72	FBC	0.18	726.54	4014
102	AIG	0.19	687.36	3578

Table 5: Top 10 financial institutions ranked according to the total sum of IN coefficients.

	Ticker	Ratio OUT	Total Sum of OUT Coefficients	Total Number of OUT Links
180	FGG	0.98	1421.37	1446
144	FFG	0.34	2278.29	6652
123	L	0.28	514.96	1815
126	AJG	0.26	479.85	1809
186	BLK	0.25	505.35	1988
177	BLW	0.23	122.82	526
196	EFX	0.23	202.69	872
23	CFR	0.22	110.98	483
158	GS	0.22	819.06	3567
193	CME	0.22	540.86	2389

Table 6: Top 10 financial institutions ranked according to the ratio of total sum of OUT coefficients to total sum of number of OUT links.

	Ticker	Ratio OUT	Total Sum of OUT Coefficients	Total Number of OUT Links
189	JNS	0.16	1694.21	10351
111	LNC	0.18	1432.52	7945
159	MS	0.18	1429.95	7756
88	FCF	0.15	1217.92	7736
99	SRCE	0.15	1215.23	7690
75	FMBI	0.17	1274.22	7466
53	NPBC	0.17	1158.28	6744
170	WDR	0.17	1158.48	6708
144	FFG	0.34	2278.29	6652
14	RF	0.18	1223.46	6533

Table 7: Top 10 financial institutions classified according to total sum of number of OUT links.

	Ticker	Ratio OUT	Total Sum of OUT Coefficients	Total Number of OUT Links
144	FFG	0.34	2278.29	6652
189	JNS	0.16	1694.21	10351
111	LNC	0.18	1432.52	7945
159	MS	0.18	1429.95	7756
180	FGG	0.98	1421.37	1446
75	FMBI	0.17	1274.22	7466
162	IVZ	0.19	1239.72	6396
115	PFG	0.20	1230.95	5917
14	RF	0.18	1223.46	6533
88	FCF	0.15	1217.92	7736

Table 8: Top 10 financial institutions classified according to total sum of OUT coefficients.

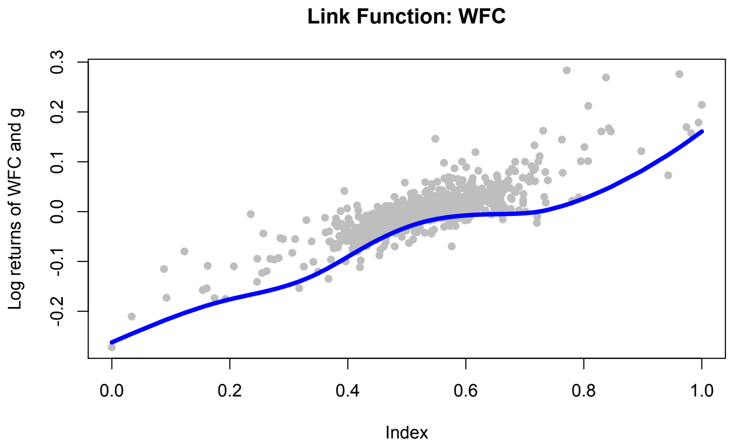

Figure 9: Average Link function vs Average Returns, September 1, 2008, Gaussian kernel, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

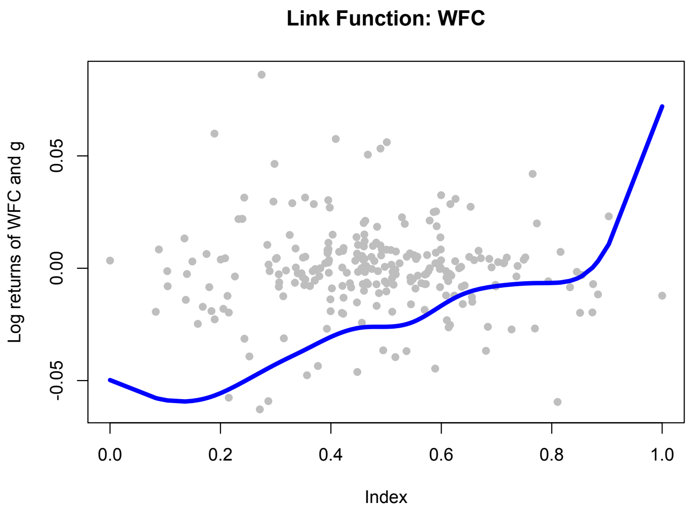

Figure 10: Average Link function vs Average Returns, June 1, 2010, Gaussian kernel, $T = 1669$, $\tau = 0.05$, window size $n = 125$.

	Ticker	Exposure
180	FGG	21.65
144	FFG	21.65
147	RDN	15.40
48	PCBC	12.11
87	PNFP	11.25
192	SLM	11.21
72	FBC	11.05
122	HIG	11.03
152	NFP	11.00
102	AIG	10.23
159	MS	10.13
17	HBAN	10.12
189	JNS	10.03
156	CIA	10.01
83	BPFH	9.97
4	BAC	9.94
14	RF	9.59
181	MKTX	9.51
9	STT	9.34
5	C	9.30
112	MBI	9.27
188	FITB	9.24
133	PL	9.23
111	LNC	9.11
34	BPOP	9.09

	Ticker	Contribution
144	FFG	33.01
189	JNS	24.83
180	FGG	21.72
111	LNC	21.13
75	FMBI	18.92
115	PFG	18.68
159	MS	18.16
14	RF	17.81
88	FCF	17.46
52	CATY	17.45
53	NPBC	16.62
99	SRCE	15.92
170	WDR	15.52
4	BAC	15.51
188	FITB	15.49
38	SUSQ	15.45
122	HIG	14.39
113	MET	14.36
5	C	14.06
181	MKTX	13.92
11	STI	13.13
117	PRU	13.08
15	KEY	13.00
30	FHN	12.56
69	MBFI	12.41

Table 9: Cumulative Contribution $\frac{\sum_{j\neq s_{m,k}} \widehat{\beta}_{j|s_{m,k}}}{\sum_{i=1}^{p}\sum_{j=1}^{p} \widehat{\beta}_{i|j}}$ (right) and Exposure Indices $\frac{\sum_{i\neq s_{m,k}} \widehat{\beta}_{s_{m,k}|i}}{\sum_{i=1}^{p}\sum_{j=1}^{p} \widehat{\beta}_{i|j}}$ (left), aggregated over windows, $T = 1669$, $\tau = 0.05$, window size $n = 125$, top 25 financial institutions.

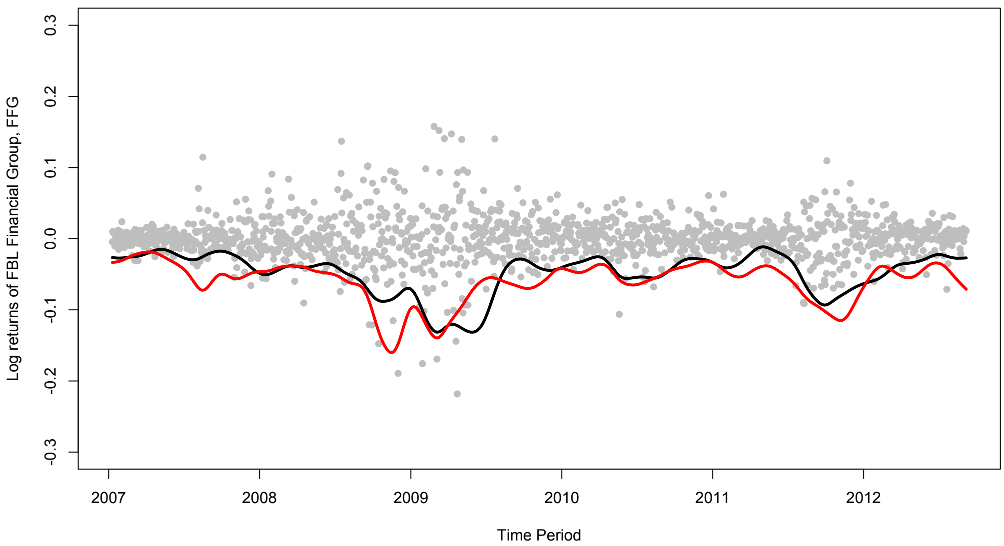

Figure 11: $\widehat{\text{VaR}}_{i,t}$ of FBL Financial Group (FFG) based on linear quantile regression on macroprudential variables (black) and on $\max\{C_{it}, E_{it}\}$ (red), $T = 1669$, $\tau = 0.05$, window size $n = 125$.

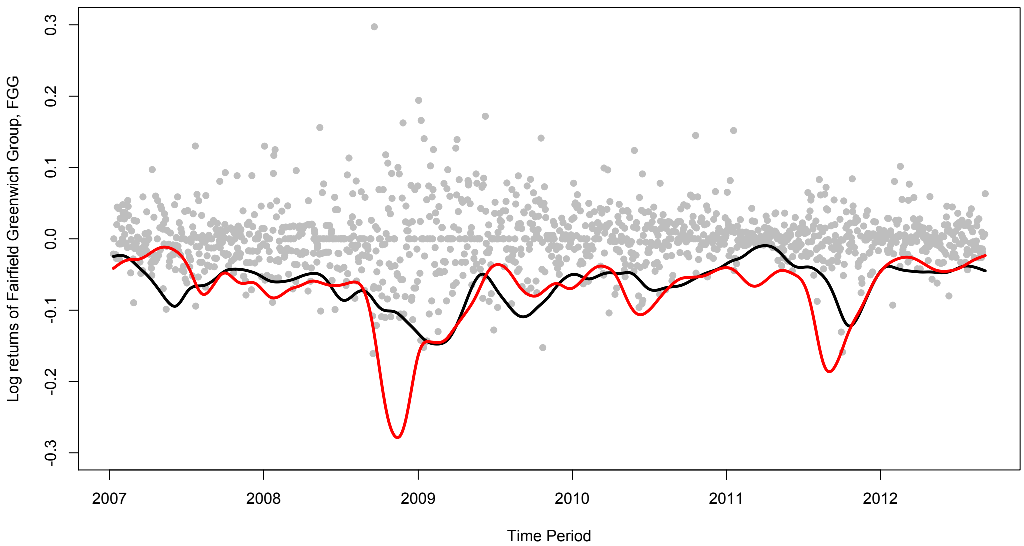

Figure 12: $\widehat{\text{VaR}}_{i,t}$ of Fairfield Greenwich Group (FGG) based on linear quantile regression on macroprudential variables (black) and on $\max\{C_{it}, E_{it}\}$ (red), $T = 1669$, $\tau = 0.05$, window size $n = 125$.

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